No Long Division of power series on Final.

12.7 # 25

$$\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$$

Test for divergence

Says $\sum$ MIGHT converge

$$\frac{n\cdot(n-1)\cdots\cdots(3)(2)(1)}{\underbrace{e^n\cdot e^n\cdots\cdots e^n\cdot e^n\cdot e^n}_{n \text{ factors}}} \xrightarrow{n\to\infty} 0 \text{ or}$$

$$e^{n^2} = \underbrace{e\cdot e\cdot\cdots\cdots e}_{n^2 \text{ factors}} \qquad (e^n)^n$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+1)!}{e^{(n+1)^2}}\cdot\frac{e^{n^2}}{n!}\right|$$

$$= \frac{n+1}{e^{(n+1)^2-n^2}} = \frac{n+1}{e^{n^2+2n+1-n^2}} = \boxed{\frac{n+1}{e^{2n+1}}} \xrightarrow{n\to\infty} 0$$

So it converges

$\sum$

$$f(x) = \sin(x) \approx T_4(x)$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x-\frac{\pi}{6}\right) - \frac{1}{4}\left(x-\frac{\pi}{6}\right)^2 - \frac{\sqrt{3}}{12}\left(x-\frac{\pi}{6}\right)^3$$

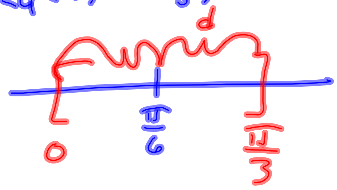
$$+ \frac{1}{48}\left(x-\frac{\pi}{6}\right)^4$$

on $\left[0, \frac{\pi}{3}\right]$ is the interval in question

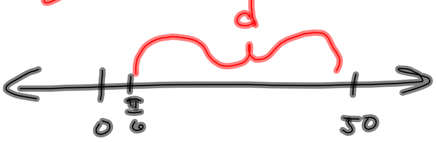
$$|R_4(x)| \le \frac{M}{5!}\left|x-\frac{\pi}{6}\right|^5$$

$$M \ge \max_{x\in[0,\frac{\pi}{3}]}\left\{|f^{(5)}(x)|\right\} = 1 = M$$

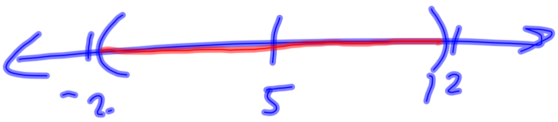
$$|R_4(x)| \le \frac{1}{5!}\left|x-\frac{\pi}{6}\right|^5 \le \frac{1}{5!}\left(\frac{\pi}{6}\right)^5$$



$[0, 50]$



$|x-5| < 7$

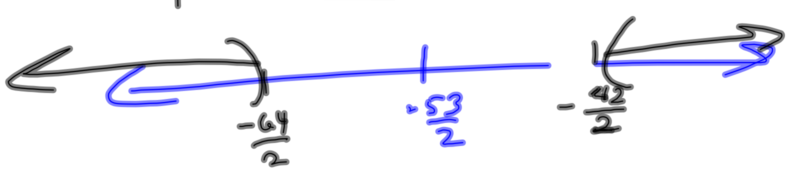


$$|2x+53| > 11$$

$$2\left|x+\frac{53}{2}\right| > 11$$

$$\boxed{\left|x+\frac{53}{2}\right|} > \frac{11}{2}$$

$$\boxed{\left|x-\left(-\frac{53}{2}\right)\right|} > \frac{11}{2}$$



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§12.10 I

$f(x) = xe^x.$

In praxis, we just do $x \sum_{k=0}^{\infty} \frac{x^k}{k!}$

$$= \sum_{k=0}^{\infty} \frac{x^{k+1}}{k!}$$

$$\sum_{n=1}^{\infty} \frac{x^n}{(n-1)!} = \sum_{n=1}^{\infty} \frac{\cancel{n}\, x^n}{\cancel{n!}\ (n-1)!} = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$$

Careful about the 1st term when integrating or differentiating series.

§12.9 #14

$f(x) = \ln(1+x)$

$f'(x) = \frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty}(-x)^n = \sum_{n=0}^{\infty}(-1)^n x^n$

So $f(x) = \int \sum_{n=0}^{\infty} (-1)^n x^n \, dx$

$= \sum_{n=0}^{\infty}(-1)^n \frac{x^{n+1}}{n+1} + C$ $\rightarrow$ $C = f(0) = 0.$

$f(0) = C$

$f(x) = \ln(1+x)$

$f(0) = \ln(1+0) = \ln(1) = 0$

$f(x) = \ln(2+x)$ $\frac{1}{2+x} = \frac{1}{2(1+\frac{x}{2})} = \boxed{\frac{1}{2}\cdot\frac{1}{1+\frac{x}{2}}}$

$f'(x) = \frac{1}{2}\sum_{n=0}^{\infty}(-1)^n \left(\frac{x}{2}\right)^n = \frac{1}{2}\sum_{n=0}^{\infty}(-1)^n \frac{x^n}{2^n}$

$\Rightarrow f(x) = \int \sum_{n=0}^{\infty}(-1)^n \frac{x^n}{2^{n+1}} dx = \sum_{n=0}^{\infty}(-1)^n \frac{x^{n+1}}{2^{n+1}(n+1)} + C$

$f(0) = \ln(2) = C$

$\ln(2+x) = \underbrace{\ln(2)}_{0^{th}\text{ term}} + \sum_{n=0}^{\infty}(-1)^n \frac{x^{n+1}}{2^{n+1}(n+1)}$

$f(x) = \frac{1}{(1+x)^2}$ $f(x) = -g'(x)$

we know $\frac{1}{1+x} = \boxed{\sum_{n=0}^{\infty}(-1)^n x^n = g(x)}$

$g(x) = (1+x)^{-1} \Rightarrow -\frac{1}{(1+x)^2} = -f(x) = g'(x)$

$f(x) = -g'(x)$

$f(x) = -\sum_{n=1}^{\infty}\boxed{(-1)^n \cdot n x^{n-1}}$

Because the derivative of the constant term is ZERO, Not $\frac{1}{x}$

$\frac{d}{dx}[5] = 0$

$f(x) = \sum_{n=0}^{\infty}(-1)^n x^n = 1 - x + x^2 - x^3 + \ldots$

$f'(x) = \overset{x^0}{-1} + 2x - 3x^2 + \ldots$

$= \sum_{n=0}^{\infty}(-1)^{n+1}(n+1)x^n$

$= \sum_{n=1}^{\infty}(-1)^n n x^{n-1}$

Trig Substitution

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