§ 12.9: #s 13, 14

Find $f(x) = \frac{1}{(1+x)^2}$

$$g(x) = \frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n$$

$$g'(x) = \frac{d}{dx}\left[(x+1)^{-1}\right] = -1(x+1)^{-2} = -\frac{1}{(1+x)^2} = -f(x)$$

$$-g'(x) = f(x)$$

$$= -\frac{d}{dx}\left[\sum_{n=0}^{\infty} (-1)^n x^n\right] = -\sum_{n=1}^{\infty} (-1)^n n x^{n-1} \text{ by formula.}$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} n x^{n-1}$$

$$= -\frac{d}{dx}\left[1 - x^1 + x^2 - x^3 + x^4 + \cdots\right]$$

$$= -\left[-1 + 2x - 3x^2 + 4x^3 + \cdots\right]$$

$$= 1 - 2x + 3x^2 - 4x^3 + \cdots$$

$$= \sum_{n=0}^{\infty} (-1)^n (n+1) x^n$$

§ 12.7 #23   Convergent?

$$\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$$

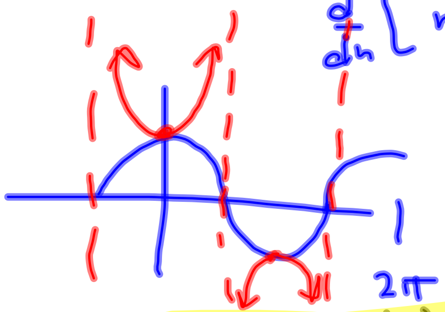
$$\lim_{n\to\infty} \tan\left(\frac{1}{n}\right) = \tan(0) = 0$$

$$\lim_{n\to\infty} \frac{\tan\left(\frac{1}{n}\right)}{\frac{1}{n}} \overset{L'H}{=} \lim_{n\to\infty} \frac{\sec^2\left(\frac{1}{n}\right)\left(-\frac{1}{n^2}\right)}{-\frac{1}{n^2}} =$$

$$\lim_{n\to\infty} \sec^2\left(\frac{1}{n}\right) = \sec^2(0) = \frac{1}{\cos^2(0)} = 1 \neq 0$$

Divergent, since $\sum \frac{1}{n}$ diverges

$$\frac{d}{dn}\left[n^{-1}\right] = -n^{-2} = -\frac{1}{n^2}$$



My idea:

$$\lim_{n\to\infty} \left| \frac{\tan\left(\frac{1}{n+1}\right)}{\tan\left(\frac{1}{n}\right)} \right| \overset{L'H}{=} \lim_{n\to\infty} \frac{\sec^2\left(\frac{1}{n+1}\right)\left(-\frac{1}{(n+1)^2}\right)}{\sec^2\left(\frac{1}{n}\right)\left(-\frac{1}{n^2}\right)}$$

$$= \lim_{n\to\infty} \left(\frac{\sec^2\left(\frac{1}{n+1}\right)}{\sec^2\left(\frac{1}{n}\right)} \cdot \frac{n^2}{(n+1)^2} \right) = 1 \cdot 1 = 1$$ sucks.

$\lim a_n$, $\lim b_n$ both exist, then → are finite

$\lim (a_n b_n) = \lim a_n \lim b_n$

Suppose $\sum b_n$ converges

and $\left|\frac{a_n}{b_n}\right| \xrightarrow{n\to\infty} 0$

Then $\sum a_n$ converges.

Suppose $\sum b_n$ diverges

and $\left|\frac{a_n}{b_n}\right| \xrightarrow{n\to\infty} 0$

Then $\sum a_n$??

Suppose $\sum b_n$ diverges &

$\left|\frac{a_n}{b_n}\right| \xrightarrow{n\to\infty} \infty$

Then $\sum a_n$ diverges.

§12.5 #12

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{e^{\frac{1}{n}}}{n}$$

$$\lim_{n\to\infty}\left(\frac{e^{\frac{1}{n}}}{n}\right)=\frac{1}{\infty}=0 \quad \text{ok}$$

Decreasing?

$$e^{\frac{1}{n+1}} < e^{\frac{1}{n}}$$

$$n+1 > n$$

$$e^{1} > e^{\frac{1}{2}} > e^{\frac{1}{3}}$$

$$\frac{e^{\frac{1}{n+1}}}{n+1} < \frac{e^{\frac{1}{n}}}{n}$$

$$y=e^{\frac{1}{x}}$$

$$\Longrightarrow y' = e^{\frac{1}{x}}\cdot -\frac{1}{x^2} < 0 \quad \forall x>0$$

So $e^{\frac{1}{x}}$ is decreasing

So $\frac{e^{\frac{1}{x}}}{x}$ is decreasing, since x is increasing.

$$\sum_{n=1}^{\infty} a_n \quad \text{where} \quad a_n=\begin{cases}\frac{1}{n} & \text{if } n \text{ is even}\\ -\frac{1}{n^2} & \text{if } n \text{ is odd}\end{cases}$$

$$= -1+\frac{1}{2}-\frac{1}{9}+\frac{1}{4}-\frac{1}{25}+\frac{1}{6}-\frac{1}{49}+\cdots$$

$1, \frac{1}{2}, \frac{1}{9}, \frac{1}{4}, \frac{1}{25}, \frac{1}{6}, \frac{1}{49}$

$$\lim_{n\to\infty}|a_n|=0$$

Alternates

But it's not decreasing.

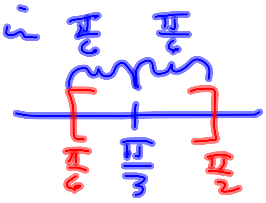
Tomorrow – $\S$ 12.9, 12.10 I
Friday – $\S$ 12.10 II, 12.11

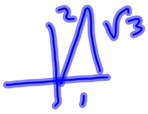
Final is Friday, 12-2

Interval of radius $\frac{\pi}{6}$, centered at $a = \frac{\pi}{3}$

$\sin x$ @ $a = \frac{\pi}{3}$ on $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

what's the max error in $T_3(x)$?





$|R_3(x)| \le$?

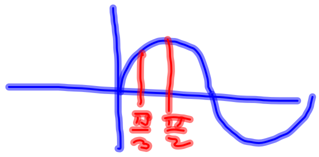
$f(x) = \sin x$ $\quad f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ $\quad |x - a| \le \frac{\pi}{6}$

$f'(x) = \cos x$ $\quad f'\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$f''(x) = -\sin(x) = -\frac{\sqrt{3}}{2}$

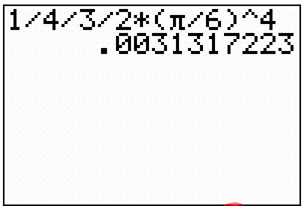
$f^{(3)}(x) = -\cos x = -\frac{1}{2}$

$f^{(4)}(x) = \sin x.$



$|f^{(4)}(x)| \le 1 = M$ on $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

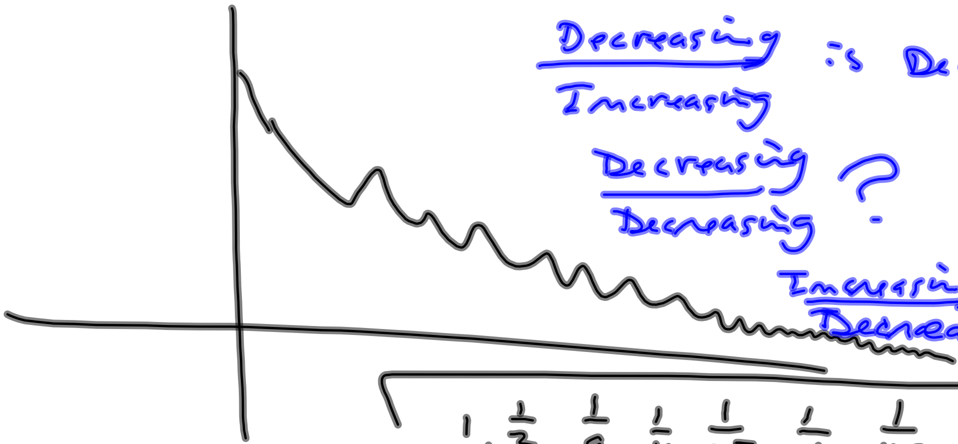
$|R_3(x)| \le \frac{M}{4!}\left|x - \frac{\pi}{3}\right|^4 = \frac{1}{4!}\left(\frac{\pi}{6}\right)^4 \approx .0031317223$


1/4/3/2*(π/6)^4
.0031317223

$$T_3(x) = \sum_{n=0}^{3} \frac{f^{(n)}\left(\frac{\pi}{3}\right)}{n!}\left(x - \frac{\pi}{3}\right)^n = \frac{\sqrt{3}}{2} + \frac{1}{2}\left(x - \frac{\pi}{3}\right) - \frac{\frac{\sqrt{3}}{2}}{2!}\left(x - \frac{\pi}{3}\right)^2 - \frac{\frac{1}{2}}{3!}\left(x - \frac{\pi}{3}\right)^3$$

Maclaurin's.

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$



$\frac{\text{Decreasing}}{\text{Increasing}}$ is Decreasing

$\frac{\text{Decreasing}}{\text{Decreasing}}$?

$\frac{\text{Increasing}}{\text{Decreasing}}$ is increas

$\boxed{1, \frac{1}{2}, \frac{1}{9}, \frac{1}{4}, \frac{1}{25}, \frac{1}{6}, \frac{1}{49}, \ldots}$

$(-1)^n \frac{e^{\frac{1}{n}}}{n}$

$$\frac{e^{\frac{1}{n}}}{n} \xrightarrow{n \to \infty} 0 \Rightarrow \text{decreasing.}$$

Newp

$\sum (-1)^n b_n$

$b_{n+1} < b_n$

Nov 30-1:11 PM

$$f'(x) = \frac{-e^{\frac{1}{x}}\cdot\frac{1}{x^2} - e^{\frac{1}{x}}}{x^2} = \frac{\underbrace{-e^{\frac{1}{x}}}_{<0}\underbrace{\left[\frac{1}{x^2}+1\right]}_{>0}}{\underbrace{x^2}_{>0}}$$

is negative.

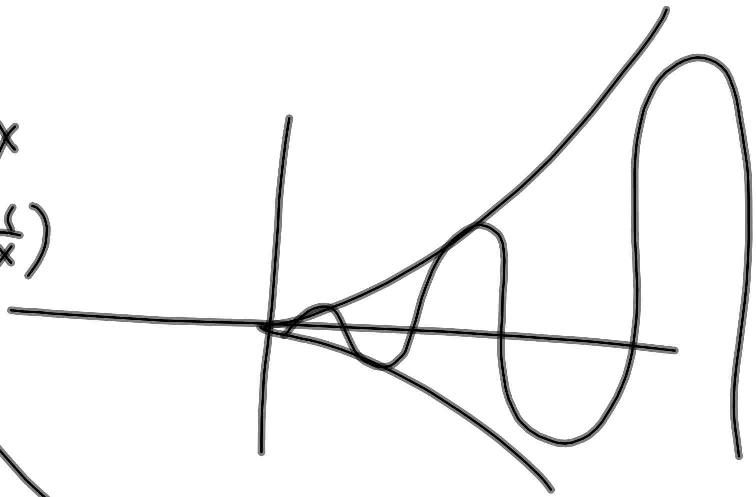
You did $\dfrac{\underbrace{-e^{\frac{1}{x}}}_{<0}\underbrace{\left(\frac{1}{x^2}-1\right)}_{<0}}{\underbrace{x^2}_{>0}}$

is positive.

so $f(x)$ is INCREASING, if $f'(x) = \dfrac{-e^{\frac{1}{x}}\left(\frac{1}{x^2}-1\right)}{x^2}$ <0

$x^2 \sin x$

$\frac{1}{x^2}\sin\left(\frac{1}{x}\right)$



$+ \sin x$

12.10 #26

$f(x) = \frac{1}{(1+x)^4} = (1+x)^{-4}$.

Expand as power series. Find Radius of Convergence.

$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 - 4x + \frac{-4(-5)}{2!}x^2 + \frac{-4(-5)(-6)}{3!}x^3 + \cdots$

$= 1 - 4x + \frac{5\cdot4}{2!}x^2 - \frac{6\cdot5\cdot4}{3!}x^3 + \frac{7\cdot6\cdot5\cdot4}{4!}x^4 - \frac{8\cdot7\cdot6\cdot5\cdot4}{5!}x^5$

$\binom{6}{3} = \frac{6!}{3!3!}$ $\binom{7}{4}$ $\binom{8}{5}$ $\frac{8!}{3!5!}$

$\binom{7}{3}$ $\binom{8}{3}$

$= \sum_{k=0}^{\infty} \binom{k+3}{3} (-1)^k x^k$ Joel

$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{\binom{n+1+3}{3}}{\binom{n+3}{3}} \frac{x^{n+1}}{x^n} \right|$

$\left| \frac{\frac{(n+4)!}{(n+1)!\cancel{3!}}}{\frac{(n+3)!}{n!\cancel{3!}}} x \right| = \frac{n+4}{n+1}|x| \xrightarrow{n\to\infty} |x|$

Need $|x| < \boxed{1 \equiv R}$

[17] THE BINOMIAL SERIES If k is any real number and $|x| < 1$, then

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots$$

Dec 2-12:13 PM

Book Ans:

$$\sum_{k=0}^{\infty} \binom{-4}{k} x^k$$

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \\ \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} \end{array}$$

$$\binom{-4}{n} = \frac{-4(-5)(-6)\cdots(-4-n+1)}{n!} =$$

$$= \frac{-4(-5)(-6)\cdots(-(n+3))}{n!}$$

$$= (-1)^n \frac{(n+3)(n+2)(n+1)}{3!} = (-1)^n \binom{n+3}{3} = (-1)^n \binom{n+3}{n}$$

$$\binom{n}{k} = \binom{n}{n-k}$$

$$k + (n-k) = n$$

$$C(10,3) = \binom{10}{3} = \frac{10!}{7!\,3!}$$

$$\binom{10}{7} = \frac{10!}{3!\,7!}$$

$$\begin{array}{c} C(10,3) \\ \shortparallel \\ {}_{10}C_3 \\ \shortparallel \\ C^{10}_3 \end{array}$$