$$\underline{\underline{\sum_{n=1}^{\infty}\left(\frac{1}{n^{1.4}}+\frac{1}{n^{1.3}}\right)}} = \sum_{\underline{n=1}}^{\infty}\frac{1}{n^{1.4}}+\sum_{\underline{n=1}}^{\infty}\frac{1}{n^{1.3}}$$ can't be done

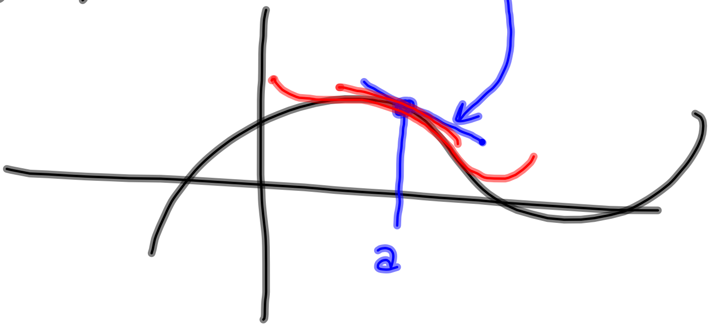
UNTIL you establish that $\sum_{n=1}^{\infty}\frac{1}{n^{1.4}}$ & $\sum_{n=1}^{\infty}\frac{1}{n^{1.3}}$ both converge.

$\sum\frac{1}{n^{1.4}}$ & $\sum\frac{1}{n^{1.3}}$ converge by p-test.

$\therefore$ $\sum\left(\frac{1}{n^{1.4}}+\frac{1}{n^{1.3}}\right)$ converges

$\sum(a_k+b_k)=\sum a_k+\sum b_k$, provided $\sum a_k$ & $\sum b_k$ both converge.

$$T_1(x) = \sum_{n=0}^{1} \frac{f^{(n)}(a)}{n!}(x-a)^n = \underline{f(a)+f'(a)(x-a)} = L_a(x)$$



If $k \in \mathbb{N}$ then

$$(1+x)^k = \sum_{n=0}^{k} \binom{k}{n} x^n$$

$$(1+x)^3 = 1 + 3x + 3x^2 + x^3$$

Maclaurin's Series for $(1+x)^3$

$$f^{(0)}(0) = 1$$

$$f^{(1)}(0) = 3(1+x)^2\Big|_{x=0} = 3$$

$$f^{(2)}(0) = 6(1+x)^1\Big|_{x=0} = 6$$

$$f^{(3)}(0) = \frac{d}{dx}(6+6x) = 6\Big|_{x=0} = 6$$

$$f^{(4)}(0) = 0 = f^{(n)}(0),\ n = 5, 6, \ldots$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = \frac{1}{0!} \cdot x^0 + \frac{3}{1!} x^1 + \frac{6}{2!} \cdot x^2 + \frac{6}{3!} x^3$$

$$= 1 + 3x + 3x^2 + x^3$$

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What if $k \notin \mathbb{N}$

Then $f^{(n)}(x)$ doesn't vanish

$\sqrt{1+x} = (1+x)^{\frac{1}{2}}$ $k=\frac{1}{2}$

a_0 $f(0)=1$

a_1 $f'(x)=\frac{1}{2}(1+x)^{-\frac{1}{2}} \Rightarrow f'(0)=\frac{1}{2}$

a_2 $f''(x)=\frac{1}{2}\cdot\left(-\frac{1}{2}\right)(1+x)^{-\frac{3}{2}} \Rightarrow f''(0)=-\frac{1}{4}$

a_3 $f^{(3)}(x)=\frac{1}{2}\cdot\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(1+x)^{-\frac{5}{2}} \Rightarrow f'''(0)=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{3}{2}$

a_4 $f^{(4)}(0)=-\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}$

a_5 $f^{(5)}(0)=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\cdot\frac{7}{2} = \frac{1}{2^n}\cdot 1\cdot3\cdot5\cdot7 = \frac{1}{2^n}\cdot 1\cdot3\cdot5\cdots(2n-3)$

$$a_n=(-1)^{n-1}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\cdots\frac{2n-3}{2} = (-1)^{n-1}\cdot\frac{1}{2^n}\cdot 1\cdot3\cdot5\cdots(2n-3)$$

$$\frac{f^{(n)}(0)}{n!}\cdot x^n = \frac{(-1)^{n-1}}{n!}\cdot\frac{1\cdot3\cdot5\cdots(2n-3)}{2^n}\,x^n$$

$$\binom{\frac{1}{2}}{0} = \frac{\frac{1}{2}}{0!\,\frac{1}{2}} = 1$$

$$\binom{k}{n} = \frac{k!}{(k-n)!\,n!}$$

$$\binom{\frac{1}{2}}{1} = \frac{\left(\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{}$$

Owie.

The idea is a vulcan mind meld between Binomial Theorem & Maclaurin expansion

$$\boxed{\binom{5}{2} = \frac{5!}{3!\,2!}} = \frac{5\cdot 4}{2!} = \frac{n(n-1)\cdots(n-k+1)}{k!} = \binom{n}{k}$$

$$\binom{\frac{1}{2}}{2} = \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}$$

$$\frac{1}{2} - 3 + 1 = -\frac{3}{2}$$

$$\frac{1}{2} - 2 + 1 = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$\binom{\frac{1}{2}}{3} = \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}$$

This is allowing us to extend the family of functions we have a "quick" power series for.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$\forall x \in \mathbb{R}$

$$\cos(3700) = \sum_{n=0}^{\infty} (-1)^n \frac{(3700)^{2n}}{(2n)!}$$

$$|x|<1 \quad \begin{cases} \dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty} x^n \\ \boxed{(1+x)^k} = \displaystyle\sum_{n=0}^{\infty} \binom{k}{n} x^n \end{cases}$$

It might take a bajillion terms.

$$\sqrt{1+x}$$

$$\frac{1}{\sqrt[3]{3+7x}} = \frac{1}{\sqrt[3]{3\left(1+\frac{7x}{3}\right)}}$$

$$= \frac{1}{\sqrt[3]{3}} \cdot \frac{1}{\sqrt[3]{1+\frac{7x}{3}}} = \frac{1}{\sqrt[3]{3}} \sum_{n=0}^{\infty} \binom{-\frac{1}{3}}{n} x^n$$

$$= \frac{1}{\sqrt[3]{3}} \left[\underbrace{1}_{n=0} + \underbrace{-\frac{1}{3}x^1}_{n=1} + \underbrace{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)x^2}_{n=2} + \underbrace{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)x^3 \cdots}_{n=3} \right]$$

$$\binom{-\frac{1}{3}}{1} = -\frac{1}{3}$$

$k = -\frac{1}{3}$ $\qquad \binom{k}{0} ?$

$$\binom{-\frac{1}{3}}{2} =$$

$\dots (k-n+1)$

$-\frac{1}{3} - 1 + 1$ $\qquad \boxed{-\frac{1}{3} - 2 + 1} = -\frac{1}{3} - 1 = -\frac{}{3}$

$$\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)$$

$\sin(x)$

$\cos(x)$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}}{\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}}$$

$$x + \frac{x^3}{3}$$

$$1 - \frac{x^2}{2} + \frac{x^4}{24} \;\Big)\; x - \frac{x^3}{6} + \frac{x^5}{120} + \cdots$$

$$-\left(x - \frac{x^3}{2} + \frac{x^5}{24} \right)$$

$$\frac{x^3}{3} - \frac{4x^5}{120}$$

$$-\left(\frac{x^3}{3} - \frac{x^5}{6} \right)$$

$x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7}$ is good guess.

$$\int_0^5 e^{-x^2}dx = \int_0^5 \sum_{n=0}^{\infty} (-x^2)^n \cdot \frac{1}{n!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad = \int_0^5 \sum_{n=0}^{\infty} (-1)^n x^{2n} \cdot \frac{1}{n!}$$

$$= \int_0^5 \left(1 - \frac{x^2}{2} + \frac{x^4}{3!} - \frac{x^6}{4!} + \frac{x^8}{5!} + \cdots\right) dx$$

$$= \left[x - \frac{x^3}{2\cdot 3} + \frac{x^5}{5\cdot 3!} - \frac{x^7}{7\cdot 4!} + \frac{x^9}{9\cdot 5!} + \cdots\right]_0^5$$

$$= \quad 5 - \frac{5^3}{2\cdot 3} + \frac{5^5}{5\cdot 3!} - \cdot$$

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