§ 12.9 #s 29,

§ 12.10 Binomials.

Generalizes the defin of $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

12.8#28

$$\frac{n!\, x^n}{1\cdot 3\cdot 5 \cdots (2n-1)}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+1)!\, x^{n+1}}{1\cdot 3\cdot 5\cdots(2n-1)\,(2(n+1)-1)} \cdot \frac{1\cdot 3\cdot 5\cdots(2n-1)}{n!\, x^n}\right|$$

$$= \left|\frac{(n+1)x}{2n+1}\right| = \left|\frac{n+1}{2n+1}\right| |x| \xrightarrow{n\to\infty} \frac{1}{2}|x| \overset{\text{want}}{<} 1$$

$R = 2$

$|x| < 2$

$-2 < x < 2$

I.C. $\frac{2^n n!}{1\cdot 3\cdot 5\cdots(2n-1)}$

$$\frac{\overbrace{2\cdot 2\cdots 2}^{n}\cdot 1\cdot 2\cdot 3\cdot 4\cdots n}{\underbrace{1\cdot 3\cdot 5\cdots(2n-1)}_{n}} = \frac{\overbrace{2\cdot 4\cdot 6\cdots 2n}^{n \text{ factors}}}{\underbrace{1\cdot 3\cdot 5\cdots(2n-1)}_{n \text{ factors}}} > 1$$

So diverges. Same for $x=-2$, since it flunks alternating series test.

(Test for divergence: $a_n \xrightarrow{n\to\infty} 0$)

$$\int x \arctan(3x)\,dx = \int x \sum_{n=0}^{\infty} \frac{(-1)^n (3x)^{2n+1}}{2n+1}\,dx$$

SEE EXAMPLE 7. we're just replacing x by 3x in arctan(x)

$$f(x) = \arctan(x) = ?$$

$$f'(x) = \frac{1}{1+x^2}$$

$$f(x) = \arctan(3x)$$

$$f'(x) = \frac{1}{1+(3x)^2}\cdot 3 = \frac{3}{1+(3x)^2} = 3\left(\frac{1}{1-(-(3x)^2)}\right)$$

$$= 3\sum_{n=0}^{\infty}\left(-(3x)^2\right)^n = 3\sum_{n=0}^{\infty}(-1)^n(3^{2n})(x^{2n})$$

$$= 3\sum_{n=0}^{\infty}(-1)^n(3x)^{2n}$$

$$\Rightarrow \arctan(3x) = 3\sum_{n=0}^{\infty}\frac{(-1)^n\cdot 3^{2n}\cdot x^{2n+1}}{2n+1}$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n(3x)^{2n+1}}{2n+1}$$ so

$$\int x\sum_{n=0}^{\infty}\frac{(-1)^n(3x)^{2n+1}}{2n+1}\,dx = \int\sum_{n=0}^{\infty}\frac{(-1)^n\cdot 3^{2n+1}\cdot x^{2n+2}}{2n+1}\,dx$$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{n-2} a^2 b^{n-2} + \binom{n}{n-1} a b^{n-1} + b^n$$

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

$$\binom{5}{3} = \frac{5!}{2!\,3!} = \frac{5 \cdot 4}{2} = 10$$

$$\binom{11}{3} = \frac{11!}{8!\,3!} = \frac{11 \cdot \overset{5}{\cancel{10}} \cdot \overset{3}{\cancel{9}}}{\cancel{3} \cdot \cancel{2}} = 165$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

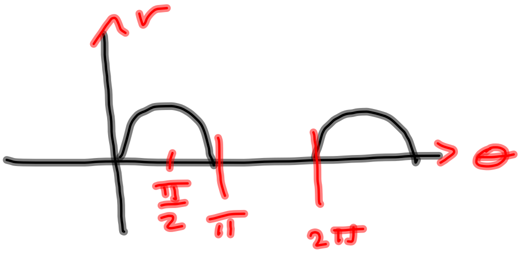
$\binom{6}{0}$ $\binom{6}{1}$ $\binom{6}{2}$ $\binom{6}{3}$ $\binom{6}{4}$ $\binom{6}{5}$ $\binom{6}{6}$

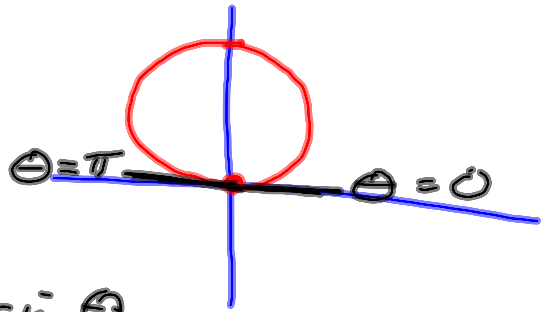
$\{1, 2, 3, 4, 5, 6\}$

12, 13, 14, 15, 16,
23, 24, 26, 34, 35, 25,
36, 45, 46, 56

$\binom{n}{k}$ = "n choose k"

4 $r = \sqrt{\sin\theta}$ $0 \le \theta \le \pi$





Limaçon

$1 + c\sin\theta$

$1 + c\cos\theta$

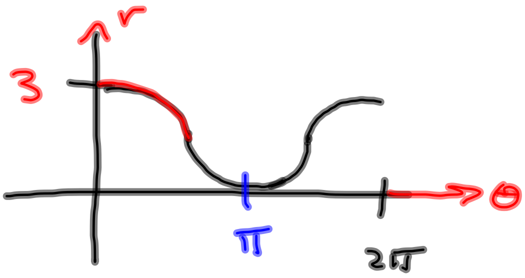
$|c| > 1$, then it has a loop.

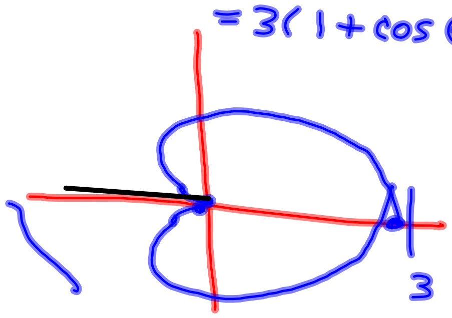
$|c| < 1$, then it has a dimple.

11.4 #.0

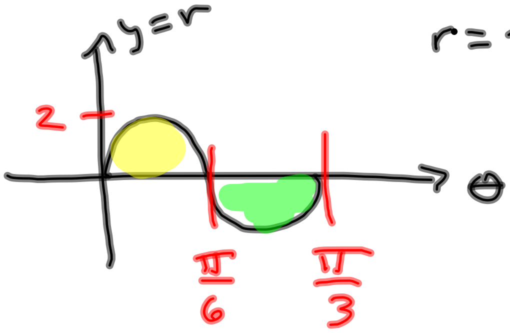
$r = 3 + 3\cos\theta = 3(\underline{1+\cos\theta})$

θ by $-\theta$: $r = 3(1+\cos(-\theta))$
$= 3(1+\cos\theta)$



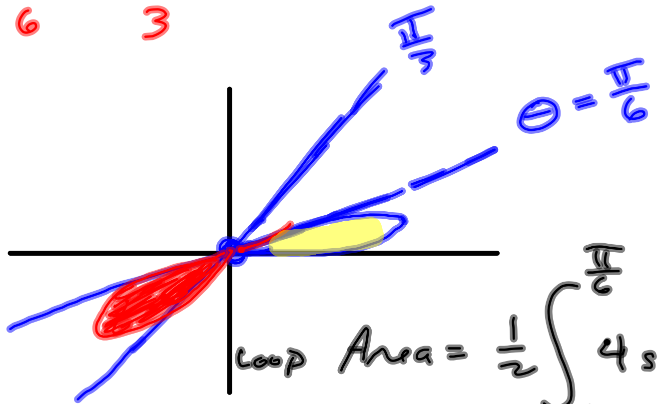


$$2 \cdot \frac{1}{2}\int_0^{\pi} \left(3(1+\cos\theta)\right)^2 d\theta$$



$r = 2\sin(6\theta)$

$\frac{2\pi}{6} = \frac{\pi}{3} =$ Period

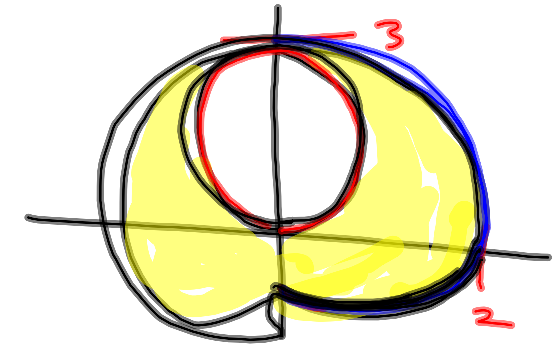


loop Area $= \frac{1}{2}\int_0^{\frac{\pi}{6}} 4\sin^2(6\theta)\,d\theta$

$= \frac{\pi}{6}$

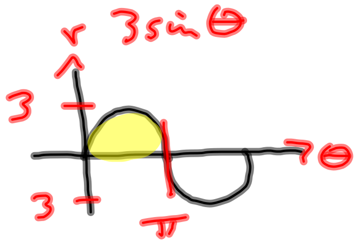
#26

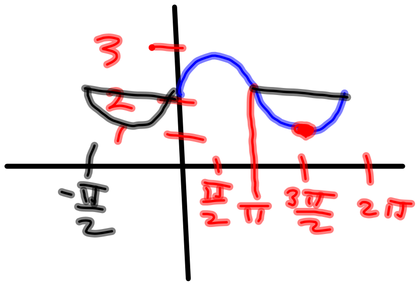
$r = 3\sin\theta$



$r = 2 + \sin\theta$

$= 2(1 + \frac{1}{2}\sin\theta)$





Outer - Inner =

$$2\left[\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(2+\sin\theta)^2\,d\theta - \frac{1}{2}\int_0^{\frac{\pi}{2}} 3\sin^2\theta\,d\theta\right]$$

$$\int_0^{\frac{\pi}{2}} + \int_{\frac{3\pi}{2}}^{2\pi}$$

$$\int_0^{\frac{\pi}{2}} + \int_{\pi}^{\frac{3\pi}{2}}$$