§12.8 #32

Power Series that converges on

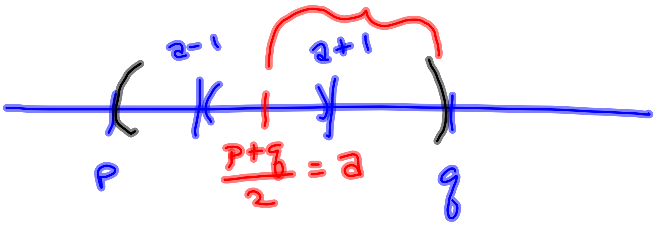
$\boxed{(p,q)}$, $\boxed{[p,q)}$, $(p,q]$, $[p,q]$

Basic

Alternating

$\int$

$$\left(\frac{x-a}{\quad}\right)^{n}$$



$$x-a = x - \frac{p+q}{2}$$

$$\frac{x-\left(\frac{p+q}{2}\right)}{\frac{q-p}{2}}$$

$$\sum_{n=0}^{\infty} \frac{(x-a)^n}{\left(\frac{q-p}{2}\right)^n} = \sum_{n=0}^{\infty} \left(\frac{x-a}{\frac{q-p}{2}}\right)^n$$

when $x=q$, I want $\dfrac{x-a}{\frac{q-p}{2}} = 1$

$x=p$, I want $\dfrac{x-a}{\frac{q-p}{2}} = -1$

$$\sum \left(\frac{x-a}{\frac{q-p}{2}}\right)^n$$

$$\frac{p-a}{\frac{q-p}{2}} = \frac{p-\left(\frac{p+q}{2}\right)}{\frac{q-p}{2}} = \frac{\frac{2p-p-q}{2}}{\frac{q-p}{2}}$$

$$= \frac{\frac{p-q}{2}}{\frac{q-p}{2}} = -1$$

$$\sum_{n=0}^{\infty} \left(\frac{2(x-2)}{q-p}\right)^n$$

$$\left| \frac{2^{n+1}(x-2)^{n+1}}{(q-p)^{n+1}} \cdot \frac{(q-p)^n}{2^n(x-2)^n} \right| = \left| \frac{2(x-2)}{q-p} \right| \overset{\text{want}}{<} 1$$

$|x-2| < \frac{q-p}{2}$, so we converge on (p,q)

$$\left| x - \frac{p+q}{2} \right| < \frac{q-p}{2}$$

$$-\frac{q-p}{2} < x - \frac{p+q}{2} < \frac{q-p}{2}$$

$$\frac{-q+p}{2} + \frac{p+q}{2} < x < \frac{q-p}{2} + \frac{p+q}{2}$$

$$p < x < q$$

Does it converge @ p or q? Nah

$x=p$: $\sum (-1)^n$

$x=q$: $\sum 1^n$

$x=p \to x-a = p - \frac{p+q}{2} =$

$= -\frac{q}{2}$

$$\sum_{n=0}^{\infty}\left(\frac{2(x-a)}{q-p}\right)^n = \sum_{n=0}^{\infty}\frac{2^n}{(q-p)^n}\cdot(x-a)^n = f(x)$$

$$\int f(x)\,dx = \sum_{n=0}^{\infty}\frac{2^n}{(q-p)^n}\frac{(x-a)^{n+1}}{n+1}$$

$$\frac{2p}{2}-\frac{p+q}{2} = \frac{2p-p-q}{2} = \frac{p-q}{2}$$

converges on (p,q), and hopefully @ p but not at q

$$\sum_{n=0}^{\infty}\left(\frac{2\,\frac{p-q}{2}}{q-p}\right)^n\frac{1}{n+1} = \sum_{n=0}^{\infty}(-1)^n\cdot\frac{1}{n+1}$$

This converges by Alternating series.

@ $x=q$: $\cdots \sum_{n=0}^{\infty}\frac{1}{n+1}$ Diverges. So this converges on $[p,q)$

$$\sum (x-a)^n \qquad |x-a|<1$$

want p to be left endpt.

q " " right "

When $x=p$, we want -1

" $x=q$, " " $+1$

$$\sum \left(\frac{2(x-2)}{9-p} \right)^n$$

$$\sum \left(\frac{2}{9-p} \right)^n \cdot \frac{(x-2)^{n+1}}{n+1}$$

$$\sum \left(\frac{2}{9-p} \right)^n \cdot \frac{(x-2)^{n+2}}{(n+1)(n+2)}$$

Argue that $9-p>2$

If not, why are we working so hard?

$$\sum \left(\frac{2}{9-p} \right)^n \cdot \frac{1}{(n+1)(n+2)}$$

p- test

$$\sum_{n=0}^{\infty} \left(\frac{2}{9-p} \right)^n \frac{(-1)^{n+2}}{(n+2)(n+1)}$$

§ 11.3

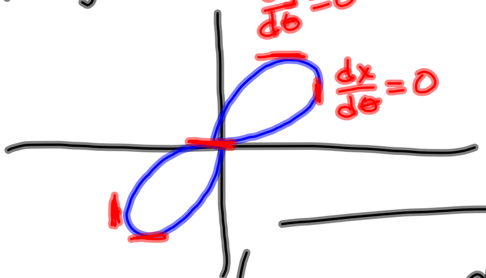
#66 $r = e^{\theta}$ is a spiral

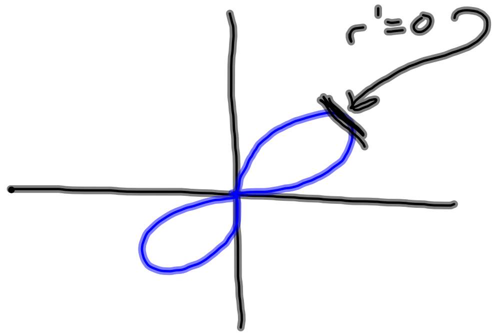
$y = r\sin\theta = e^{\theta}\sin\theta$

$r' = e^{\theta}$ is a way to capture maximum radii.

No help

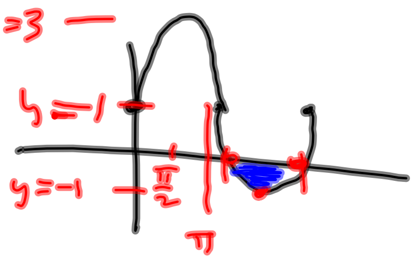
Asking for





$$\boxed{\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}} =$$

$1 + 2\sin\theta$



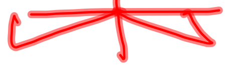
when does it have a loop?

$\frac{1}{2}\int f(\theta)^2 \, d\theta$

11.4 question with graph provided.

$2\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$

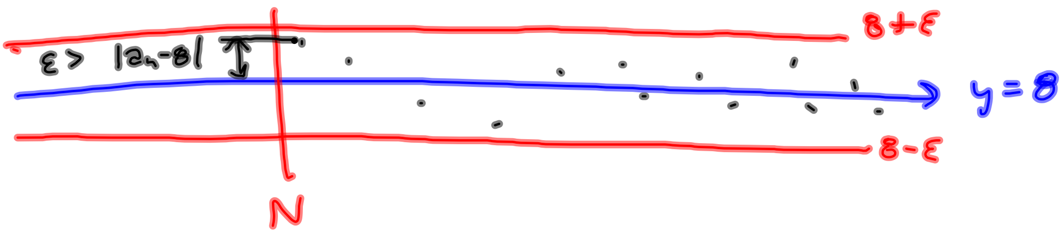


§12.1

$$\lim_{n\to\infty} a_n = 8$$

Given any $\varepsilon > 0$, there is an $N \in \mathbb{N}$ such that

$|a_n - 8| < \varepsilon$ whenever $n > N$.



$\sum b \cdot \underset{\approx}{r}^n$ converges for $-1 < r < 1$

$\sum b \cdot (x-a)^n$ " " $-1 < x-a < 1$

want $r = -1$ when $x = p$

" $r = +1$ " $x = q$

$\left(x - \frac{p+q}{2}\right)^n$ at $x = p$, this is $\left(\frac{p-q}{2}\right)^n$

$$\boxed{\text{want } (-1)^n \text{ @ } x = p}$$

Want $(1)^n$ @ $x = q$

so $\left(\dfrac{\frac{p-q}{2}}{?}\right)^n = (-1)^n$?

$$\left(\dfrac{\frac{p-q}{2}}{\frac{q-p}{2}}\right)^n = (-1)^n$$

Nov 21-12:47 PM

Applications of this theory include:

• Approximating a function by using its Taylor polynomial (Early calculators used Taylor polynomials to do 'most all their trig. calculations).

• Approximating a definite integral by integrating the first handful of terms of the Taylor series corresponding to its integrand.

• Evaluating Limits by expressing all or part of the expression in terms of a series.

$f(x) = \int_0^x \frac{1}{1+t^5}\,dt$ can be thought of as approximating a function. We plugged in $x=.2$ in the 11/18/11 notes for 12.9.

We also built a power series for cos(x), centered at $a=0$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

Using Taylor's Series:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$= \underbrace{f(a) + f'(a)(x-a)} + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

Nov 30-10:41 AM