12.6 # 11

$$\sum_{n=1}^{\infty} \frac{(-1)^n e^{\frac{1}{n}}}{n^3}$$

Alternating: All we need is $b_n \xrightarrow{n\to\infty} 0$

$b_n = \frac{e^{\frac{1}{n}}}{n^3} < \frac{e}{n^3} \xrightarrow{n\to\infty} 0$

$\boxed{\text{Conditionally Convergent}}$

$\boxed{\text{I think } e^{\frac{1}{n}} < e}$

I wasn't sure at first. so I looked @

Is enough for what we did, below.

$f(x) = e^{\frac{1}{x}} \Longrightarrow f'(x) = -\frac{1}{x^2} e^{\frac{1}{x}}$ is negative.

So $f(n) = e^{\frac{1}{n}}$ is decreasing $\forall n > 0$

$e^{\frac{1}{n}} < e^{\frac{1}{1}} = e$

Absolute:

was inconclusive $\left|\frac{a_{n+1}}{a_n}\right| \xrightarrow{n\to\infty}$ Hold the bus:

$\frac{e^{\frac{1}{n}}}{n^3} < \frac{e}{n^3}$, so $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^3}$ compares

favorably with $\sum_{n=1}^{\infty} \left|\frac{e}{n^3}\right| = e \sum_{n=1}^{\infty} \left|\frac{1}{n^3}\right|$, which passes p-test.

$\therefore$ converges absolutely.

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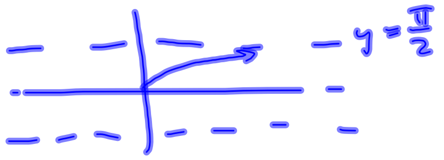
12.6 # 15

$$\sum \frac{(-1)^n \arctan(n)}{n^2}$$



$$b_n = \frac{\arctan(n)}{n^2} < \frac{\frac{\pi}{2}}{n^2}$$

$\frac{\pi}{2}\sum \frac{1}{n^2}$ converges by p-test.

So $\sum \frac{(-1)\arctan(n)}{n^2}$ converges absolutely by direct comparison to $\sum \frac{\pi/2}{n^2}$.

(24) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{\ln(n)^n}$

$y = n^{\frac{1}{n}}$

$\ln(y) = \frac{1}{n}\ln(n) = \frac{\ln(n)}{n}$

$$\sqrt[n]{\left|\frac{n}{\ln(n)^n}\right|} = \frac{n^{\frac{1}{n}}}{\ln(n)} \xrightarrow{n\to\infty} 0$$

$\xrightarrow[L'H]{n\to\infty} \frac{\frac{1}{n}}{1} = \frac{1}{n} \xrightarrow{n\to\infty} 0$

So absolutely convergent. so $\lim\limits_{n\to\infty} n^{\frac{1}{n}} = 1$

Another, less efficient, way to get a handle on the $n^{\frac{1}{n}}$

$y = x^{\frac{1}{x}}$

$\ln(y) = \frac{1}{x}\ln(x)$

$$\frac{y'}{y} = -\frac{1}{x^2}\cdot \ln(x) + \frac{1}{x}\cdot\frac{1}{x} = -\frac{1}{x^2}\left[\ln(x) - 1\right]$$

$$y' = \underbrace{-\frac{1}{x^2}\underbrace{\left[\ln(x)-1\right]}_{>0 \text{ if } x>e} x^{\frac{1}{x}}}_{<0 \text{ if } x>e.}$$

$n^{\frac{1}{n}}$ decreasing for $n>e$, $n\geq 3$

$\boxed{\text{Thus } n^{\frac{1}{n}} < 3^{\frac{1}{3}}}$ for all $n>3$

so $\frac{n^{\frac{1}{n}}}{\ln(n)} < \frac{3^{\frac{1}{3}}}{\ln(n)} \xrightarrow{n\to\infty} 0$

Claim: If $\sum a_n$ & $\sum b_n$ converge, then prove / disprove $\sum a_n b_n$ converges.

$$\sum_{n=1}^{2} a_n \sum_{n=1}^{2} b_n \qquad \sum_{n=1}^{2} a_n b_n$$

$$(a_1+a_2)(b_1+b_2) = a_1b_1 + a_1b_2 + a_2b_1 + a_2b_2 \qquad = a_1b_1 + a_2b_2$$

$$\sum a_n \sum b_n \le \sqrt{\sum |a_n b_n|^2} \quad \text{Cauchy}$$

$$\int f(x)dx \int g(x)dx \le \sqrt{\int (f(x)g(x))^2}$$

$b_n, a_n \ge 0$

$\sum a_n$ converges. $\Rightarrow a_n \xrightarrow{n\to\infty} 0$ → eventually $a_n b_n \le b_n$

Let $\varepsilon > 0$ Then $\exists N \in \mathbb{N} \ni$

$|a_n - 0| < \varepsilon$, i.e.

$a_n < \varepsilon$

Let $\varepsilon = 1$

Then $\exists N \in \mathbb{N} \ni a_n < 1 \ \forall n \ge N$.

So $a_n b_n < 1 \cdot b_n = b_n \ \forall n \ge N$.

So $\sum_{k=N}^{\infty} a_k b_k$ compares favorably to $\sum_{k=N}^{\infty} b_k$ so $\sum_{k=N}^{\infty} a_k b_k$ converges, by Direct Comparison to $\sum_{k=N}^{\infty} b_k$

Since the N-tail converges, the whole thing converges

↳ EVENTUALLY.

§12.2

$$\sum_{n=1}^{\infty}(x-4)^n = \sum_{n=1}^{\infty}\underbrace{(x-4)}_{a}(x-4)^{n-1} \qquad \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

$$= (x-4)\sum_{n=1}^{\infty}(x-4)^{n-1} = \frac{x-4}{1-(x-4)} = \frac{x-4}{5-x} \qquad \boxed{\sum_{k=1}^{n} ar^{k-1} = \frac{a(1-r^n)}{1-r}}$$

$$\int (t^2+5t-7)\,dx = (t^2+5t-7)\int dx$$

$$\boxed{\sum_{k=0}^{n} ar^k = \frac{a(1-r^{n+1})}{1-r}}$$

Need $|r| = |x-4| < 1$

$-1 < x-4 < 1 \qquad x-4 < 1$ AND $x-4 > -1$

$\boxed{3 < x < 5} \qquad x < 5$ AND $x > 3$

$\{x \mid 3 < x < 5\} = \underline{(3,5)}$

Check Endpts

$\frac{d}{dx} \sum c_k x^k$ tends to rob the endpts.

$f(x)$ on $[a,b)$, $f'(x)$ on (a,b)

$\int \sum c_k x^k \, dx$ " " give you the endpts

$f(x)$ on $[a,b)$, $\int f(x)\,dx$ on $[a,b]$

There are a few interesting surprises these results give us. We can really expand our flight envelope quite a bit.

15–18 Find a power series representation for the function and determine the radius of convergence.

$\boxed{15.}$ $f(x) = \ln(5 - x)$

You gotta be kiddin' me!!! Ahhhh. We *do* know the derivative of this function! And its derivative has a nice power series representation. So to find what we're looking for, we first TAKE the derivative of f , find the power series for f, and then we take the term-by-term *antiderivative* to get the final answer.

And think of the possibilities for estimating integrals of some uglier functions that have warm 'n' fuzzy power series representations... You just integrate the power series term by term, and use as many terms as you need to achieve the desired accuracy.

More Spinoffs:

23–26 Evaluate the indefinite integral as a power series. What is the radius of convergence?

24. $\displaystyle\int \frac{\ln(1-t)}{t}\,dt$

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27–30 Use a power series to approximate the definite integral to six decimal places.

See also: Example 8.

27. $\displaystyle\int_0^{0.2} \frac{1}{1+x^5}\,dx$

Find a power series for

$$f(x) = \frac{7}{(1-x)^2}$$

$$f'(x) = -2\cdot 7(1-x)^{-1}(-1) = \frac{14}{1-x} = 14\cdot\frac{1}{1-x}$$

$$= 14\sum_{n=0}^{\infty} x^n \implies$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\boxed{f(x) = \left(14\sum_{n=0}^{\infty}\frac{x^{n+1}}{n+1}\right)} + C = 14\sum_{n=0}^{\infty}\frac{x^{n+1}}{n+1} + 7$$

and this will be given by $f(0)$.

$$f(0) = \frac{7}{1^2} = 7$$

$$f(x) = \frac{7x^5}{(1-x)^2} = x^5\left[14\sum_{n=0}^{\infty}\frac{x^{n+1}}{n+1} + 7\right]$$

$$= 14\sum_{n=0}^{\infty}\frac{x^{n+6}}{n+1} + 7x^5$$

$\arctan(x) = f(x)$ Same deal

$f'(x) = \frac{1}{1+x^2}$, etc.

§12.9

See Example 7

$f(x) = \arctan(x)$

$f(x) = \ln(1+x)$

$$f'(x) = \frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n$$

$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} + f(0) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$$

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$$\boxed{\int_0^1 e^{x^2}dx} = \int_0^1 \sum_{n=0}^{7} \frac{f^{(n)}(.5)}{n!}(x-.5)^n dx$$

$$e^{x^2} = f(x)$$

$$f'(x) = 2xe^{x^2}$$

$$f''(x) = 4x^3e^{x^2} + 2e^{x^2}$$

$$f'''(x) = 12x^2e^{x^2} + 4x^3 \cdot 2xe^{x^2}$$

$f'(.5)$

$f''(.5)$

$f'''(.5)$