12.7 Strategies for Series

$$\sum_{n=1}^{\infty} ar^{n-1} = \sum_{n=0}^{\infty} ar^n$$

$$\sum_{k=0}^{n} a \cdot r^k = \frac{a(1-r^{n+1})}{1-r}$$

$|r| < 1 \Longrightarrow$

$$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$$

① p-series

② geometric Series

③ Anything "similar" to ① & ② that you can compare

(i) Directly $a_n \le b_n$

(ii) In the limit. $\frac{a_n}{b_n} \xrightarrow{n\to\infty} c \neq 0$

④ $\lim_{n\to\infty} a_n \neq 0$ Diverges

⑤ Alternating series.

⑥ Factorials <u>ratio test</u>

⑦ $a_n = (b_n)^n$, where you know something about b_n <u>root test</u>

⑧ $a_n = f(n)$, where $\int^{\infty} f(x)\,dx$, when the integral is manageable.

12.4 #45

$\sum a_n \longrightarrow$, $a_n \geq 0 \ \forall n$,

Is $\sum \sin(a_n) \longrightarrow$?

$\lim_{n\to\infty} \sin(\frac{1}{n}) = \lim_{h\to 0} \sin(h)$

change of variable

$$\boxed{\frac{\sin(\frac{1}{n})}{\frac{1}{n}} = \frac{\sin(u)}{u} \xrightarrow{u \to 0} 1}$$

, so $\sum \sin(\frac{1}{n})$ has same convergence properties as $\sum \frac{1}{n}$

$\frac{d}{dx}[\sin(x)]$:

$$\frac{\sin(x+h) - \sin(x)}{h} = \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$= \sin(x)\left(\frac{\cos h - 1}{h}\right) + \frac{\sin h \cos x}{h} \xrightarrow{h \to 0} \cos x$$

$\to 0$ as $h \to 0$

$\to 1$ as $h \to 0$

§ 12.5 #22

$\boxed{\text{Calc. } S_{10}}$ for $\sum \frac{(-1)^{n-1}}{n^3}$ $\boxed{\text{use } a_{11}}$

$\boxed{\text{Estimate the error.}}$

2 things for error estimate.

(1) Integral $R_n \le \int_n^\infty f(x)\,dx$

(2) Alternating Series $R_n \le a_{n+1}$

$$\sum_{n=k+1}^{\infty} (-1)^n a_n \le |a_{k+1}|$$

$$\sum_{n=1}^{10} \frac{(-1)^{n-1} \cdot 1}{n^3}$$

$$\frac{14420574181}{16003008000}$$

$evalf(\%)$

$$0.9011164764$$