Solve the Initial Value Problem

$$y'' + 2y' - 24y = 0$$

Subject to $y(0) = 1$, $y'(0) = 3$

12.6 ABSOLUTE CONVERGENCE AND THE RATIO AND ROOT TESTS

1 DEFINITION A series Σa_n is called **absolutely convergent** if the series of absolute values $\Sigma |a_n|$ is convergent.

$$\sum \frac{1}{n^2}$$

2 DEFINITION A series Σa_n is called **conditionally convergent** if it is convergent but not absolutely convergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[4]{n}} \qquad \sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{1}{n}$$

3 THEOREM If a series Σa_n is absolutely convergent, then it is convergent.

EXAMPLE 3 Determine whether the series

$$\sum_{n=1}^{\infty} \frac{\cos n}{n^2} = \frac{\cos 1}{1^2} + \frac{\cos 2}{2^2} + \frac{\cos 3}{3^2} + \cdots$$

is convergent or divergent.

■ Figure 1 shows the graphs of the terms a_n and partial sums s_n of the series in Example 3. Notice that the series is not alternating but has positive and negative terms.

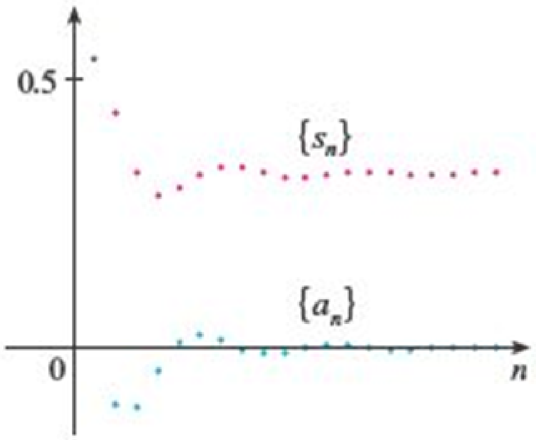


FIGURE 1

$$\left|\frac{\cos(n)}{n^2}\right| \le \frac{1}{n^2}$$

Since $\sum \frac{1}{n^2}$ converges,
the sum $\sum \left|\frac{\cos(n)}{n^2}\right|$
converges, so
$\sum \frac{\cos(n)}{n^2}$ converges
absolutely, hence
converges.

PROOF

(i) The idea is to compare the given series with a convergent geometric series.

Not sure where this came from, but I think it's a remark about the Ratio Test.

THE RATIO TEST

(i) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and therefore convergent).

Go back to your plan. #13 §12.6

Sorry. Space Case.

$$4^{2n+1} = 4^{2n}\cdot 4^1 = 4\cdot 4^{2n} = 4\cdot(4^2)^n = 4\cdot 16^n$$

$$\sum \frac{10^n}{(n+1)4^{2n+1}}$$

$$\frac{10^n}{(n+1)4^{2n+1}} = \frac{10^n}{(n+1)\cdot 4\cdot 16^n} = \frac{1}{4(n+1)}\cdot\frac{10^n}{16^n} = \frac{1}{4(n+1)}\cdot\left(\frac{10}{16}\right)^n = \frac{1}{4(n+1)}\cdot\left(\frac{5}{8}\right)^n$$

By direct comparison to $\sum \frac{1}{4}\cdot\left(\frac{5}{8}\right)^n$, we know it converges.

Can you use a Direct Comparison to $\sum \frac{1}{4}\cdot\left(\frac{5}{8}\right)^n$ for $\frac{1}{4(3n-8)}\cdot\left(\frac{5}{8}\right)^n$?

Need $0 \le a_n \le b_n$ to compare

$$\sum_{n=1}^{\infty}\frac{1}{4(3n-8)}\left(\frac{5}{8}\right)^n = \sum a_n \text{ to } \sum b_n = \sum_{n=1}^{\infty}\frac{1}{4}\left(\frac{5}{8}\right)^n$$

The first 2 terms are negative. So direct comparison to THEM doesn't work. Need to start with $n=3$, but from then on, $0 \le a_n \le b_n$.

That's part of "eventually"

$$\sum \frac{\ln(n)}{n^p}$$

For what powers of p does this converge?

$p<0$: $\lim_{n\to\infty} \frac{\ln(n)}{n^p} = \infty$

$0<p\leq 1$: $\ln(n) > 1$ if $n>2$

so $\frac{\ln(n)}{n^p} > \frac{1}{n^p}$ & $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges if $0<p\leq 1$

what about $p>1$?

Test for divergence:

$$\lim_{n\to\infty} \frac{\ln(n)}{n^p} \overset{L'H}{=} \frac{\frac{1}{n}}{pn^{p-1}} = \frac{1}{pn \cdot n^{p-1}} = \frac{1}{pn^p}$$

$\xrightarrow{n\to\infty} 0$, so no easy out, here.

Beth suggests Integral Test.

Applies to decreasing functions.

We need to know that $\frac{\ln(n)}{n^p}$ is a decreasing function of n.

$f(x) = \frac{\ln(x)}{x^p} \Longrightarrow$ I forgot Quotient Rule!

$$\frac{\frac{1}{x}\cdot x^p - \ln(x)\, p x^{p-1}}{x^{2p}} = \frac{x^{p-1} - p\ln(x)x^{p-1}}{x^{2p}}$$

$$= \frac{\overset{+}{x^{p-1}}\left(1-p\ln(x)\right)}{x^{2p}} \to +$$

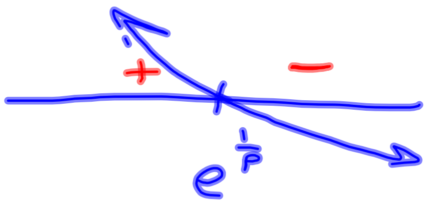
Look @ sign pattern for $1-p\ln(x)$

$$y=-p\ln(x)+1 \overset{\text{SET}}{=} 0$$

$$p\ln(x)=1$$

$$\ln(x)=\frac{1}{p}$$

$$x=e^{\frac{1}{p}}$$



It says $\frac{\ln(n)}{n^p}$ is decreasing for $n>e^{\frac{1}{p}}$

<u>Eventually</u>, it's decreasing and the integral test applies.

Next time: I'll do Ratio Test proof.

· And Root Test examples.

THE RATIO TEST

(i) If $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and therefore convergent).

(ii) If $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1$ or $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

(iii) If $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$, the Ratio Test is inconclusive; that is, no conclusion can be drawn about the convergence or divergence of $\Sigma\, a_n$.

34. Let Σa_n be a series with positive terms and let $r_n = a_{n+1}/a_n$. Suppose that $\lim_{n\to\infty} r_n = L < 1$, so Σa_n converges by the

Ratio Test. As usual, we let R_n be the remainder after n terms, that is,

$$R_n = a_{n+1} + a_{n+2} + a_{n+3} + \cdots$$

(a) If $\{r_n\}$ is a decreasing sequence and $r_{n+1} < 1$, show, by summing a geometric series, that

$$R_n \leqslant \frac{a_{n+1}}{1 - r_{n+1}}$$

(b) If $\{r_n\}$ is an increasing sequence, show that

$$R_n \leqslant \frac{a_{n+1}}{1 - L}$$