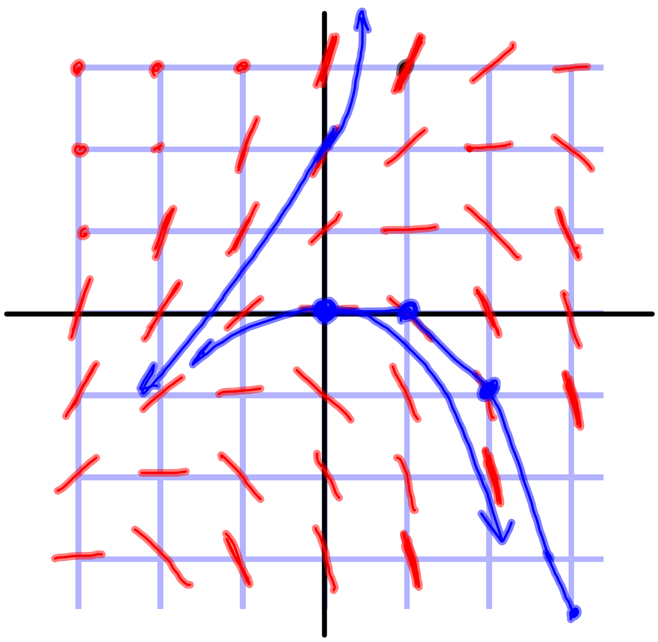

$y' = y - x$

$$y_{n+1} = y_n + hF(x_n, y_n)$$
$$= y_n + 1 \cdot (x_n - y_n)$$

$\Delta x = h$

$$y'' - 3y' - 28y = 0$$

$$D = \frac{d}{dx} \qquad y'(0) = y(0) = 1$$

$$D^2 = \frac{d^2}{dx^2}$$

$$D^2y - 3Dy - 28y = 0$$

$$(D^2 - 3D - 28)y = 0$$

$(D-7)(D+4) = 0$ or $y = 0$ → Nope $y(0) = 1$ can't do it.

$$D = -4, 7$$

$$y = c_1e^{-4x} + c_2e^{7x}$$

$$y' = -4c_1e^{-4x} + 7c_2e^{7x}$$

$y(0) = c_1 + c_2 = 1 \Longrightarrow c_1 = 1 - c_2$

$y'(0) = -4c_1 + 7c_2 = 1 \rightarrow -4(1 - c_2) + 7c_2 = 1$

$$-4 + 4c_2 + 7c_2 = 1$$

$$11c_2 = 5$$

$$c_2 = \frac{5}{11} \Longrightarrow c_1 = 1 - \frac{5}{11} = \frac{6}{11}$$

$$y = \frac{6}{11}e^{-4x} + \frac{5}{11}e^{7x}$$

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$$\boxed{y'' - 3y' - 28y = 0}$$

e^x is as steep as it is tall

$y' = ky$

$y = ce^{kx}$

$y' = kce^{kx}$

$\$$ $y = ce^{kx}$ is a solution. Find k

$y' = kce^{kx}$

$y'' = k^2ce^{kx}$

$0 = D^2 - 3D - 28$ is the Characteristic Polynomial

$$y'' - 3y' - 28y = k^2ce^{kx} - 3kce^{kx} - 28ce^{kx} \stackrel{SET}{=} 0$$

$$\Rightarrow \quad ce^{kx}\left[k^2 - 3k - 28\right] = 0$$

$ce^{kx} = 0$ only if $c = 0$. Boring

or $k^2 - 3k - 28 = 0 \Rightarrow$ ... $k = 7, -4$. See?

$(k-7)(k+4)$

$y' = 7y$

$y = ce^{7x}$

$y' = -4y$

$y = ce^{-4x}$

Rog, Terry

The "system." The "establishment" has you waiting 'til I fill your empty cup and then trying to race through homework in one sitting.

12.7 : Put 12.1–12.6 together

Awesome

$$S = \sum_{k=1}^{\infty} a_k \qquad S_n = \sum_{k=1}^{n} a_k, \quad R_n = \sum_{k=n+1}^{\infty} a_k$$

<u>You Know</u> $S = S_n + R_n$

$$R_n \le \int_n^{\infty} f(x)\,dx, \text{ where } f(n) \equiv a_n$$

$$R_n \ge \int_{n+1}^{\infty} f(x)\,dx$$

Terms are positive*, decreasing*; f(x) is positive*, decreasing*

*Eventually.

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(32) Σ

S_{10} for $\sum_{n=1}^{\infty} \frac{1}{n^4}$ (a) Estimate error and then estimate S from S_{10}

(b) Use (3) with $n=10$ to get an improved estimate

(c) Find $n \ni |S_n - S| < .00001$ $\rightarrow |R_n|$

$$S_{10} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots + \frac{1}{10^4} = \frac{43635917056897}{40327580160000}$$

$$\approx \boxed{1.082036583 \approx S \text{ using } S_{10}}$$

$$S_{10} + \int_{11}^{\infty} \frac{1}{x^4}\,dx \le \underbrace{S_{10} + R_{10}}_{S} \le S_{10} + \int_{10}^{\infty} \frac{dx}{x^4}$$

$$\int_{11}^{\infty} \frac{dx}{x^4} = \lim_{t \to \infty} \int_{11}^{t} \frac{dx}{x^4} = \lim_{t \to \infty} \left[\frac{x^{-3}}{-3} \right]_{11}^{t} = \lim_{t \to \infty} \left[\frac{t^{-3}}{-3} - \frac{11^{-3}}{-3} \right]$$

$$= \frac{11^{-3}}{3}$$

Likewise $\int_{10}^{\infty} \frac{dx}{x^4} = \frac{10^{-3}}{3}$

$$\underbrace{\frac{43635917056897}{40327580160000} + \frac{11^{-3}}{3}} \le S \le \underbrace{\frac{43635917056897}{40327580160000} + \frac{10^{-3}}{3}}$$

Take the average

(b)

$$\frac{\frac{43635917056897}{40327580160000} + \frac{11^{-3}}{3} + \frac{43635917056897}{40327580160000} + \frac{10^{-3}}{3}}{2}$$

Want error less than .00001

$$\text{Error} = R_n \le \underbrace{\int_n^\infty \frac{dx}{x^4} < .00001}$$

Guarantees $R_n \le .00001$

$$\lim_{t\to\infty}\left[-\frac{x^{-3}}{3}\right]_n^t = \frac{n^{-3}}{3} = \frac{1}{3n^3} < .00001$$

Tomorrow, quiz on problem like E3 #6.