

§ 12.1 Sequences

ⓓ A sequence is a list of #s written in a particular order.

$$a_1, a_2, a_3, \dots$$

Alternate A sequence is a function from \mathbb{N} into \mathbb{R} , i.e., $f: \mathbb{N} \rightarrow \mathbb{R}$

$$a_1 = f(1), a_2 = f(2), \dots$$

This alternate lets us use what we know about functions from \mathbb{R} into \mathbb{R} to say things about sequences.

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

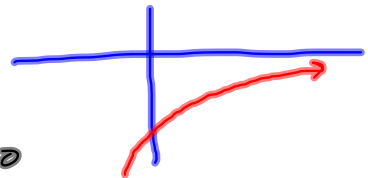
Notation $\{a_1, a_2, a_3, \dots\} = \{a_n\} = \{a_n\}_{n=1}^{\infty}$

Examples $\left\{ \frac{\ln(n)}{n} \right\}$

Application of "Alternate"

Fact: $a_n = \frac{1}{n}$ converges to zero as $n \rightarrow \infty$

Note: $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$



$$\left\{ (-1)^n \cdot \frac{1}{n} \right\} = \left\{ -1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, \dots \right\}$$

$$\left\{ (-1)^{n+1} \cdot \frac{1}{n} \right\} = \left\{ 1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \dots \right\} = \left\{ (-1)^{n-1} \cdot \frac{1}{n} \right\}$$

Fun puzzles Find a_n in general:

(#10) $\left\{ 1, \frac{1}{2}, \frac{1}{9}, \frac{1}{27}, \dots \right\}$

(#13) $1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27},$

$n=1$ $n=2$ $n=3$ $n=4$

D $\{a_n\}$ has limit L and we write

$$\lim_{n \rightarrow \infty} a_n = L \quad \text{OR} \quad a_n \rightarrow L \text{ as } n \rightarrow \infty$$

$$\text{OR} \quad a_n \xrightarrow{n \rightarrow \infty} L$$

Means

if we can make a_n as close to L as we like by taking n sufficiently large.

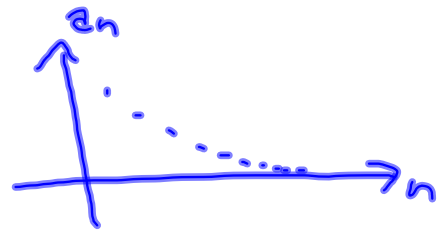
(And every a_n after that will also be within the desired tolerance.)

If $a_n \xrightarrow{n \rightarrow \infty} L, \{a_n\}$ converges.

N diverges.

E Harmonic Sequence

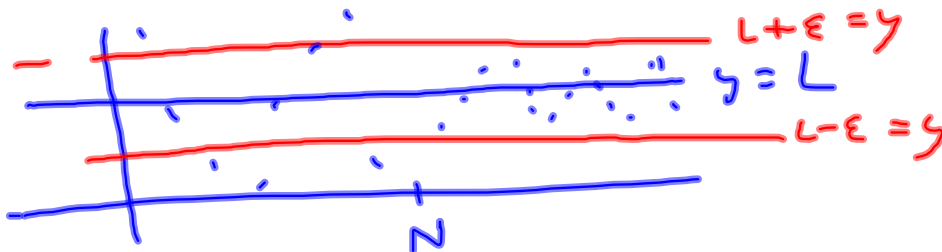
$$\left\{ \frac{1}{n} \right\} \quad \cdot \quad \frac{1}{n} \xrightarrow{n \rightarrow \infty} 0$$



D2 $\{a_n\}$ has limit L , i.e., $a_n \xrightarrow{n \rightarrow \infty} L$

iff

$$\forall \epsilon > 0, \exists N \in \mathbb{N} \exists \text{ if } n > N \text{ then } |a_n - L| < \epsilon.$$



$$(a_n \xrightarrow{n \rightarrow \infty} L) \iff (\forall \epsilon > 0 \exists N \in \mathbb{N} \exists \text{ if } n > N \text{ then } |a_n - L| < \epsilon)$$

#15 1st 6 terms of $\{a_n\} = \left\{ \frac{n}{2n+1} \right\}$

Does it converge?

i.e., does $f(x) = \frac{x}{2x+1}$ have a HORIZONTAL ASYMPTOTE.

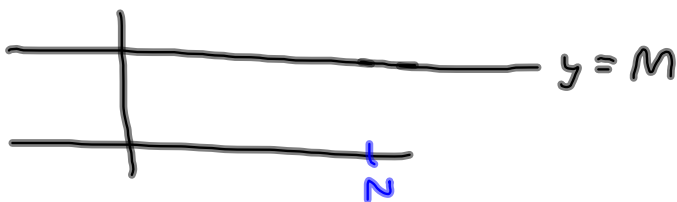
T3 $f(x) \xrightarrow{x \rightarrow \infty} L$ & $a_n = f(n)$

Then $a_n \xrightarrow{n \rightarrow \infty} L$

D5 $a_n \xrightarrow{n \rightarrow \infty} \infty$ means if you give me

$M > 0$, I can find $N \in \mathbb{N}$ such that

$a_n > M$ for every $n > N$.



Limit Laws $c \in \mathbb{R}$

$$\lim (a_n + b_n), \lim (a_n - b_n), \lim (c a_n),$$

$$\lim (a_n b_n), \lim \left(\frac{a_n}{b_n} \right), \lim (a_n^p)$$

$$\lim (a_n^p) = \left(\lim (a_n) \right)^p$$

[T7] If $a_n \xrightarrow{n \rightarrow \infty} L$ & f is cont^s, then

$$\lim_{n \rightarrow \infty} f(a_n) = f(L)$$

$$= f\left(\lim_{n \rightarrow \infty} a_n\right)$$

1, 2, 3, 5, 7 - 45 ODDS, 47, 49, 51, 53, (56?) 57,
61, 63, (67!) **Honest Effort.**

$$5 \bmod 2 = 1$$

$$5 \bmod 2 = 0$$

$$n=1 \quad \frac{1}{1}$$

$$n=3 \quad \frac{1}{2}$$

$$n=5 \quad \frac{1}{3}$$

$$a_n = \begin{cases} \frac{1}{k} & \text{if } n = 2k - 1 \\ \frac{1}{2k} & \end{cases}$$

A STAB at #44 - Above and beyond the instructions, we're trying to formulate a definition for a_n

$$n=2 \quad \frac{1}{2}$$

$$n=4 \quad \frac{1}{4}$$

$$n=6 \quad \frac{1}{6}$$

$$n=8 \quad \frac{1}{8}$$

$$n=10 \quad \frac{1}{10}$$

$$10 =$$