Find all points of intersection

$r = \sin\theta,\ r = \sin(2\theta)$

$\sin\theta = \sin 2\theta = 2\sin\theta\cos\theta$ (Identity)

$2\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(2\cos\theta - 1) = 0$

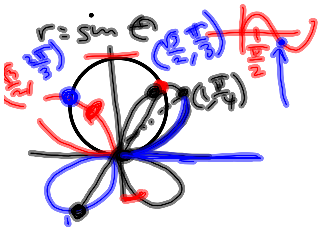
$\sin\theta = 0$ or $2\cos\theta - 1 = 0$

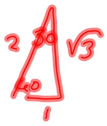
$\theta = 0, \pi, \cancel{2\pi}$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

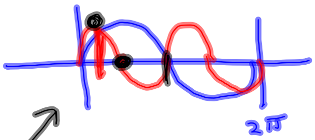






$2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$r = \sin\theta$



$\frac{3\pi}{2} + \frac{\pi}{6} = \frac{10\pi}{6} = \frac{5\pi}{3}$

$\left(\frac{\sqrt{3}}{2}, \frac{2\pi}{3}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{5\pi}{3}\right)$

These representations aren't unique.

$\frac{2\pi}{3} + \pi = \frac{2\pi + 3\pi}{3} = \frac{5\pi}{3}$

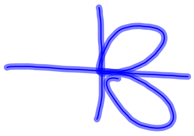
$r = \sin\theta = r(\theta)$

$r(-\theta) = \sin(-\theta) = -\sin\theta$ so it's symmetric about... polar axis.

Symmetry

$r(\theta) = r(-\theta)$ Symmetric about polar axis.

That means this:

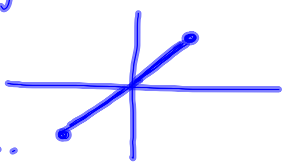


Replace $r$ by $-r$ & no change:

That's symmetry

$r^2 = \sin^2\theta$ Symmetry thru the origin.

$(-r)^2 = r^2 \ldots$

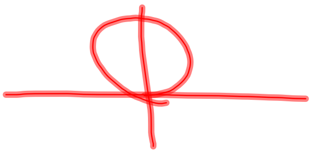


Replace $\theta$ by $\pi - \theta$ & no change

$\sin(\pi - \theta) = -\sin(\theta - \pi)$

$-\sin\theta$

$-\sin(\theta - \pi) = \sin\theta$



Symmetry about $\theta = \frac{\pi}{2}$

Arc length

$r = f(\theta)$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$x = f(\theta)\cos\theta$

$y = f(\theta)\sin\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

T 11.2.6 says

$$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \quad \text{, so....}$$

$$\left(\frac{dx}{d\theta}\right)^2 = f'(\theta)^2\cos^2\theta \underbrace{-2f'(\theta)\cos\theta f(\theta)}_{2r'\cos\theta\, r\sin\theta} + f(\theta)^2\sin^2\theta$$

use $r$ in place of $f(\theta)$:

$$= r'^2\cos^2\theta \underbrace{-2r'\cos\theta\, r}_{-2r'\cos\theta\, r\sin\theta} + r^2\sin^2\theta$$

Boo-Boo

Likewise:

$$\left(\frac{dy}{d\theta}\right)^2 = r'^2\sin^2\theta + 2r'\sin\theta\, r\cos\theta + r^2\cos^2\theta$$

$$\Rightarrow \left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = r'^2(\cos^2\theta + \sin^2\theta) + r^2(\cos^2\theta + \sin^2\theta) = r'^2 + r^2$$

$$= \left(\frac{dr}{d\theta}\right)^2 + r^2$$

So $\sqrt{(x')^2 + (y')^2}\, d\theta = \sqrt{(r')^2 + r^2}\, d\theta$

r

Punchline:

$$\int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Oct 27-12:52 PM

$r = e^{\theta}$ $\quad 0 \le \theta$