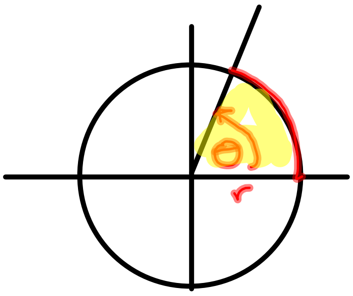


Arc length $= r\theta$
so it is proportional to θ

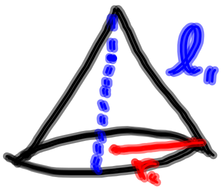
Area $= \frac{1}{2}r^2\theta$ is proportional to θ

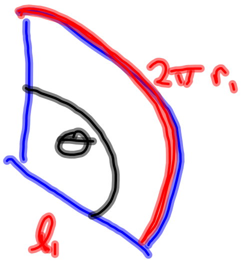
$$\frac{\text{Arc length}}{\text{angle}} = \frac{\text{Circumference}}{2\pi} = \frac{\text{Area of circle}}{\pi r^2} = \frac{\text{Area of sector}}{\theta}$$

$$\frac{\pi r^2}{?} = \frac{2\pi}{\theta} \Longrightarrow \frac{r^2}{?} = \frac{2}{\theta} \Longrightarrow \frac{1}{2}r^2\theta = ? = \text{Area of sector.}$$

? → Area of sector

Surface area of Cone





$l_1\theta = 2\pi r_1 =$ Arc length

Area of the sector is

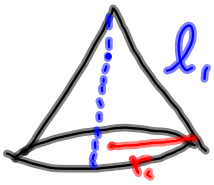
$$\frac{1}{2}l_1^2\theta = \frac{1}{2}l_1 \cdot l_1\theta$$
$$= \frac{1}{2}l_1 \cdot 2\pi r_1$$
$$\frac{1}{2}l_1^2\theta = \pi l_1 r_1 \longrightarrow$$
$$\theta = \frac{2\pi l_1 r_1}{l_1^2} = \frac{2\pi r_1}{l_1}$$

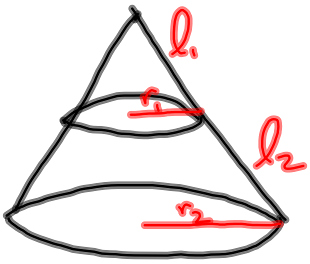
We need this.

Area of this cone is

$$\boxed{\pi l_1 r_1}$$

Oct 25-12:45 PM





Area of the bottom piece.
→ Frustum

Area of this cone is

$$\boxed{\pi l_1 r_1}$$

Area = Whole cone − Little cone

$$\boxed{\pi(l_1+l_2)r_2 - \pi l_1 r_1}$$

↳ Pick up HERE, tomorrow, and then revert back to 11.3-4 stuff, hopefully near to area part.

$\pi(l_1+l_2)r_2 - \pi l_1 r_1$

$= \pi l_1 r_2 + \pi l_2 r_2 - \pi l_1 r_1$

$= \pi\left((r_2-r_1)l_1 + l_2 r_2\right)$

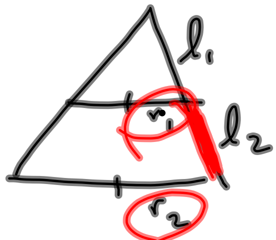
$= \pi\left(-l_2 r_1 + l_2 r_2\right)$

$= \pi(r_1-r_2)l_2$

Now, $(r_2-r_1)l_2 = l_2 r_1 \Rightarrow$

$\text{Area} = \pi(l_2 r_1 + l_2 r_2)$

$= \pi \underbrace{l_2}_{ds}(r_1+r_2) = 2\pi l_2\left(\frac{r_1+r_2}{2}\right)$



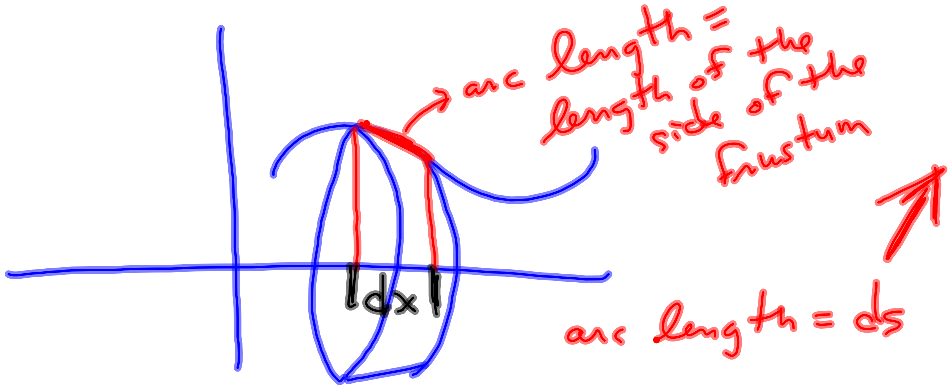
Similar triangles

$\frac{l_1+l_2}{r_2} = \frac{l_1}{r_1}$

$l_1 r_1 + l_2 r_1 = l_1 r_2$

$l_1 r_1 - l_1 r_2 = -l_2 r_1$

$l_1(r_1 - r_2) = -l_2 r_1$



$$2\pi \ell_2\left(\frac{r_1+r_2}{2}\right) = 2\pi\, ds\left(\frac{f(x_1)+f(x_2)}{2}\right)$$

$= 2\pi \overline{f(x)}\, ds$, where $\overline{f(x)}$ = average of $f(x_1)$ & $f(x_2)$

If dx is small, then $f(x_1)$ & $f(x_2)$ are pretty close.

We can argue that $\exists\, x^* \in (x_1, x_2)$ such that

$f(x^*) = \frac{f(x_1)+f(x_2)}{2}$ (Think Mean value Theorem)

we can pick any $x \in (x_1, x_2)$ and $f(x) \approx f(x_1) \approx f(x_2)$

$r_1 \approx r_2 \approx f(x)$. This gives

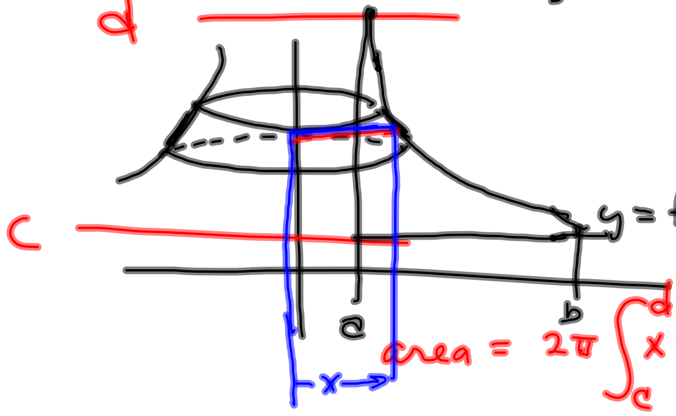
$$2\pi f(x)\, ds = 2\pi f(x)\sqrt{1+(f'(x))^2}\, dx$$

From this, we have Area $= 2\pi \int_a^b f(x)\, ds$

$= 2\pi \int_a^b y\, ds$

Oct 26-12:14 PM

To revolve about y-axis



area = $2\pi \int_c^d x\, ds$

If given $y=f(x)$

$$2\pi \int_a^b x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

If given $x=g(y)$

$$2\pi \int_c^d x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

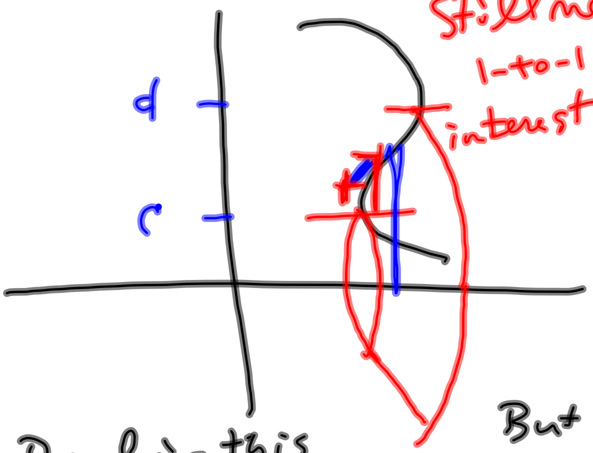
Requires getting $x=g(y)$

$\int y\, ds$ about x-axis

given $y=f(x)$, then $\int f(x)\, ds$ → $\sqrt{\cdots}\, dx$

given $x=g(y)$, then $\int y\, ds$ → $\sqrt{\cdots}\, dy$

$\int x\, ds$ about y-axis



Still need to be 1-to-1 on the interval of interest

$x=g(y)$, not 1-to-1

$\Longrightarrow$ you <u>can't</u> easily find $y=f(x)$ by inverting $x=g(y)$

But locally, any smooth function is 1-to-1.

Revolving this about x-axis is a bear

$$2\pi\int y\,ds = 2\pi\int_c^d y\sqrt{1+(g'(y))^2}\,dy$$

Up-Shot: You don't have to find the inverse function when you're revolving $x=g(y)$ about x-axis, nor do you need to invert $y=f(x)$ when revolving about the y-axis.

Revolve $\qquad x \in \qquad \int x\,ds$

$f(x) = x^2+1$ on $[1,2]$ about x-axis $\qquad \int y\,ds$

$y = x^2+1$

$y-1 = x^2$

$\sqrt{y-1} = x$ (for $x \geq 0$)

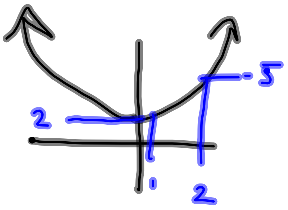
$$2\pi \int y\,ds = 2\pi \int_1^2 (x^2+1)\sqrt{1+(2x)^2}\,dx$$

about y-axis

$$2\pi \int x\,ds = 2\pi \int_1^2 x\sqrt{1+(2x)^2}\,dx = 2\pi \int_2^5 \sqrt{y-1}\sqrt{1+\tfrac{1}{4}(y-1)}\,dy$$

$x = y^2+1$

$\frac{1}{2}\int f(\theta)^2\,d\theta$



$\frac{dx}{dy} = \frac{1}{2}(y-1)^{-\frac{1}{2}}$

$2\pi \int x\,ds$ y-axis