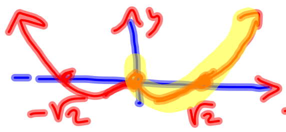
§11.2 #32

$x = t^2 - 2t$, $y = \sqrt{t}$ $\quad t \geq 0$, $y \geq 0$

I immediately eliminate the parameter, because I can:

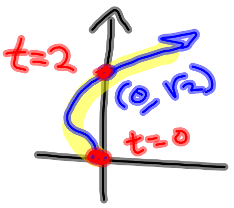
$y = \sqrt{t} \Rightarrow t = y^2 \Rightarrow x = (y^2)^2 - 2(y^2) = y^2(y^2-2) = y^2(y-\sqrt{2})(y+\sqrt{2})$

If this were $y = x^2(x^2-2) = x^2(x-\sqrt{2})(x+\sqrt{2})$



$y = x^4 - 2x^2 = x^2(x^2-2)$

$x = y^2(y^2-2)$



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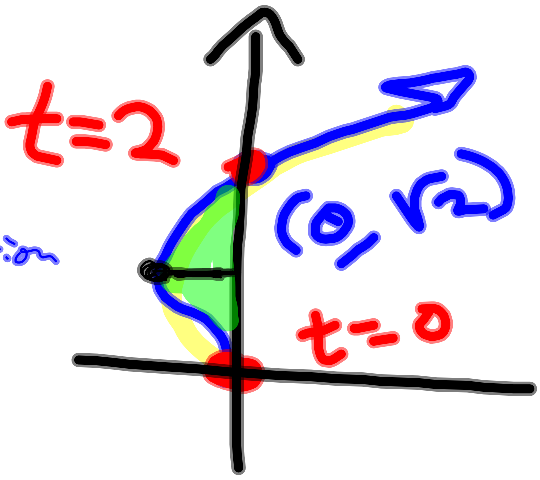


Looks like an $x = g(y)$ situation

In this case

$$\text{Area} = \int_{y=0}^{y=\sqrt{2}} g(y)\,dy$$

$= x$, but left of y-axis is "negative height" and will result in negative area.

$\int |x|\,dy$

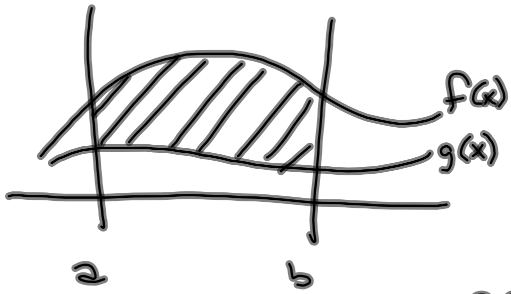
$$\boxed{\int_{y=0}^{y=\sqrt{2}} (x_R - x_L)\,dy} = \int_0^{\sqrt{2}} (0 - y^2(y^2-2))\,dy = -\int_0^{\sqrt{2}} y^2(y^2-2)\,dy$$

$$= -\int_{t=0}^{t=2} (t^2-2t)\left(\tfrac{1}{2} t^{-\frac{1}{2}} dt\right)$$ & the rest is Calc I.

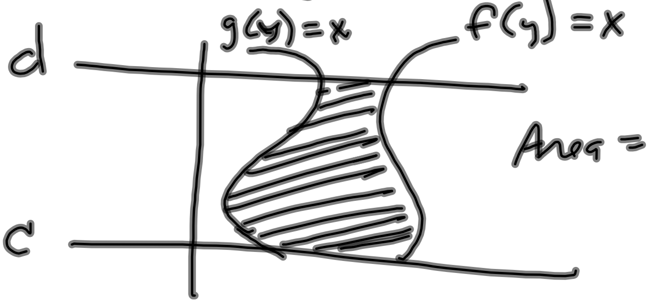
$y = \sqrt{t}$

$\Rightarrow dy = \frac{dt}{2\sqrt{t}} = \frac{1}{2} t^{-\frac{1}{2}} dt$

Oct 25-12:12 PM



$$\text{Area} = \int_a^b \big(f(x) - g(x)\big)\,dx$$



$$\text{Area} = \int_c^d \big(\underbrace{f(y)}_{x_R} - \underbrace{g(y)}_{x_l}\big)\,dy$$

$$x_R - x_l$$

§11.3 Derivatives piece of it

Recall §11.1-ish:

$y = g(t),\ x = f(t) \Longrightarrow$

$\frac{d}{dx}\left[y\right] = \frac{dy}{dx} = \dfrac{\frac{d}{dt}[y]}{\frac{d}{dt}[x]}$, and concavity is found by formal substitution of $\frac{dy}{dx}$ for y, we obtain

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{d}{dt}[x]} = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{dx}{dt}}$$

New!

$\S$ $y = r\sin\theta$, $x = r\cos\theta$, & that $r = f(\theta)$

Then $y = f(\theta)\sin\theta$, $x = f(\theta)\cos\theta$ and

$(fg)' = f'g + fg'$ $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

At the pole, $r = 0 = f(\theta) \Longrightarrow \frac{dy}{dx} = \tan\theta$. cool.

Pick up here, tomorrow on 11.4.

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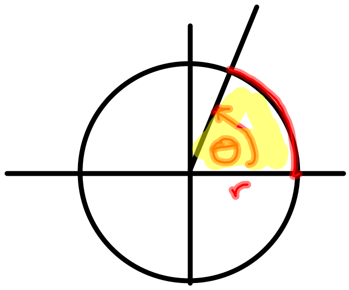


Arc length $= r\theta$
so it is proportional to θ

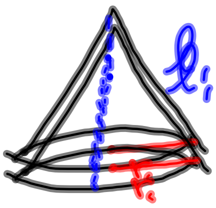
Area $= \frac{1}{2}r^2\theta$ is proportional to θ

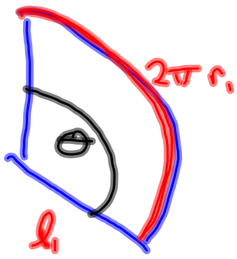
$$\frac{\text{Arc Length}}{\text{angle}} = \frac{\text{Circumference}}{2\pi} = \frac{\text{Area of circle}}{\pi r^2} = \frac{\text{Area of sector}}{\theta}$$

$$\frac{\pi r^2}{?} = \frac{2\pi}{\theta} \Longrightarrow \frac{r^2}{?} = \frac{2}{\theta} \Longrightarrow \frac{1}{2}r^2\theta = ? = \text{Area of sector.}$$

? → Area of sector

Surface area of Cone





$l_1\theta = 2\pi r_1 =$ Arc Length

Area of the sector is

$$\frac{1}{2}l_1^2\theta = \frac{1}{2}l_1 \cdot l_1\theta$$
$$= \frac{1}{2}l_1 \cdot 2\pi r_1$$

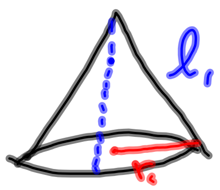
$$\frac{1}{2}l_1^2\theta = \pi l_1 r_1 \longrightarrow$$

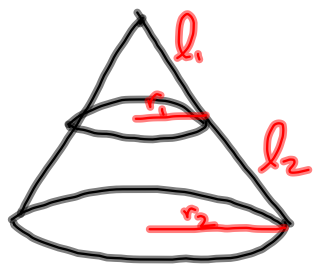
$$\theta = \frac{2\pi l_1 r_1}{l_1^2} = \frac{2\pi r_1}{l_1}$$

we need this.

Area of this cone is

$$\boxed{\pi l_1 r_1}$$





Area of the bottom piece.
↳ Frustum

Area of this cone is

$$\boxed{\pi l_1 r_1}$$

Area = Whole cone − Little cone

$$\boxed{\pi(l_1+l_2)r_2 - \pi l_1 r_1}$$

↳ Pick up HERE, tomorrow, and then revert back to 11.3-4 stuff, hopefully near to area part.