10.1

Going from how it moves to where it is.

$y' = x,\ y(0) = 1$ $\qquad$ $\frac{1}{2}x^2 + 1$

$\frac{dy}{dx} = x$

When possible, integrate to solution.

$dy = x\,dx$

$\int dy = \int x\,dx$

$y = \frac{1}{2}x^2 + C$

$y(0) = \frac{1}{2}(0)^2 + C = 1$

$\Longrightarrow C = 1$

$\Longrightarrow y = \frac{1}{2}x^2 + 1$

Linear DE's 10.5

$y' + P(x)\,y = f(x)$

$e^{\int p(x)\,dx}$ is the integrating factor.

$(xy)' = \frac{d}{dx}[xy]$

$= y + xy'$

See nasty derivation in 10.5 notes.

11.2 #10 Find an equation of the tangent(s) to the curve @ the given point.

$(-1,1)$ is the point.

$x = \cos(t) + \cos(2t)$ $\qquad$ $y = \sin(t) + \sin(2t)$

$\frac{dx}{dt} = -\sin(t) - 2\sin(2t)$ $\qquad$ $\frac{dy}{dt} = \cos(t) + 2\cos(2t)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos(t) + 2\cos(2t)}{-\sin(t) - 2\sin(2t)}$$

at any given (x, y) on the graph.

what values of t will give us $(-1,1)$?

$x = -1$

$\cos(t) + \cos(2t) = -1$

$\cos(t) + 2\cos^2(t) - 1 = -1$

$2\cos^2(t) + \cos(t) = 0$

$2u^2 + u = 0$

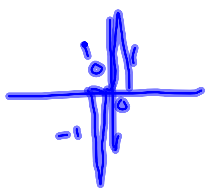
$u(2u+1) = 0$

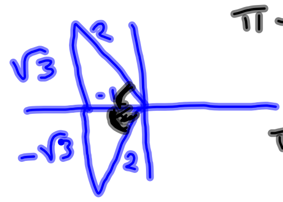
$u = 0$ or $2u + 1 = 0$

$\cos(t) = 0$ or $2\cos(t) + 1 = 0$

$\cos(t) = -\frac{1}{2} \Rightarrow t = \frac{2\pi}{3}, \frac{4\pi}{3}$

$t = \frac{\pi}{2}, \frac{3\pi}{2}$





$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

Now see which, if any, of these t-values results in $y = \sin(t) + \sin(2t) = 1$

$t = \frac{\pi}{2}$

$\sin(\frac{\pi}{2}) + \sin(2 \cdot \frac{\pi}{2}) = 1 + 0 = 1$ ✓

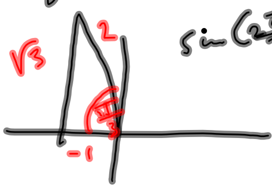
We'll pick up on this.

§11.2 I #s 1, 2, 4, 5, 9, 11, 18, 25, 29 Monday.

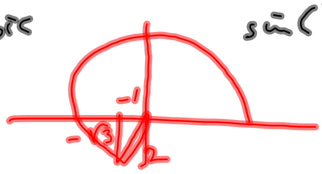
$y = \sin\left(\frac{3\pi}{2}\right) + \sin(3\pi) = -1 + 0 = -1$ Nope

$y = \sin\left(\frac{2\pi}{3}\right) + \sin\left(\frac{4\pi}{3}\right) = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = 0$ Nope

$\sin\left(\frac{2\pi}{3}\right)$ pic



$\sin\left(\frac{4\pi}{3}\right)$ pic

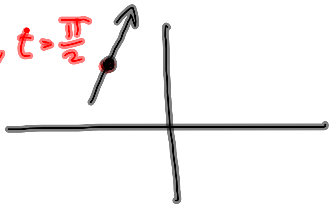


$y = \sin\left(\frac{4\pi}{3}\right) + \sin\left(\frac{8\pi}{3}\right) = -\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = 0$

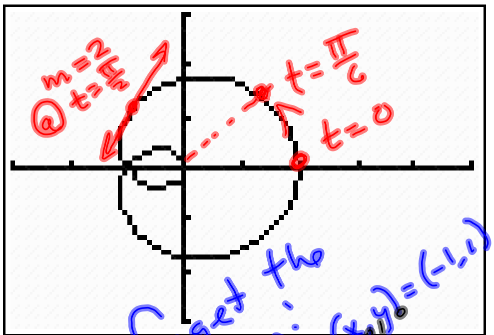
$\frac{8\pi}{3} = \frac{6\pi + 2\pi}{3} = 2\pi + \frac{2\pi}{3}$

@ $t = \frac{\pi}{2}$, $y = 1$, $x = -1$

$(x, y) = (-1, 1)$, $t = \frac{\pi}{2}$



$$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{2}} = \left.\frac{\cos(t) + 2\cos(2t)}{-\sin(t) - 2\sin(2t)}\right|_{t=\frac{\pi}{2}} = \frac{\cos\left(\frac{\pi}{2}\right) + 2\cos(\pi)}{-\sin\left(\frac{\pi}{2}\right) - 2\sin(\pi)} = \frac{0 - 2}{-1 - 0} \boxed{= 2 = m}$$



$x(t) = \cos(t) + \cos(2t)$

$y(t) = \sin(t) + \sin(2t)$

t	x	y
0	2	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}+1}{2}$	$\frac{\sqrt{3}+1}{2}$
$\frac{\pi}{2}$	-1	1

Don't forget the original question: TANGENT Line @ $(x_0, y_0) = (-1, 1)$

$y = m(x - x_0) + y_0$

$y = 2(x - (-1)) + 1$

$= 2(x + 1) + 1$

$= 2x + 2 + 1$

$= 2x + 3$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Don't just stack

$$\frac{d^2y}{dt^2} \dots$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} \neq \frac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}}$$

Yes No

$$\frac{d}{dx}\left[\frac{dy}{dt}\right] = \frac{d}{dt}\left[\frac{dy}{dx}\right] \ ?$$

Speculation