§10.5 #s 1–6, 8, 13, 17, 20, 23, 24, 26*

* 26 is a 2^{nd}-order equation

Test Monday Tomorrow & Friday Review

§9.3 OMIT. Put it on Final.

§10.5 Linear Differential Equations

AMPERSAND

$$(xy)' = \frac{d}{dx}[xy]$$

$$\& \int (xy)'dx = \int \frac{d}{dx}[xy]\,dx = \int d(xy) = xy + C$$

$$\int (xy)'dx = xy + C$$

$$\frac{d}{dx}[xy] = y + xy' = (xy)'$$

Consider $\underline{y' + \frac{1}{x}y = 2}$

$$\underbrace{xy' + y}_{(xy)'} = 2x$$

$$(xy)' = 2x$$

$$\int (xy)'dx = \int 2x\,dx$$

$$xy = x^2 + C$$

$$y = x + \frac{C}{x}$$

Multiplying both sides by x made the left hand side into a recognizable derivative.
That allowed us to integrate to a solution.
"x" was our integrating factor.

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Linear D.E.:

$$y' + P(x)y = Q(x)$$

Want $I(x)$ so that

$$\underbrace{I(x)\left[y' + P(x)y\right]}_{(I(x)y)'} = I(x)\,Q(x)$$

If we have such an $I(x)$, then

$$I(x)\,y = \int (I(x)y)'\,dx = \int I(x)\,Q(x)\,dx$$

$$y = \frac{\int I(x)\,Q(x)\,dx}{I(x)}$$

Let's look @ $I(x)\left[y' + P(x)y\right]$

$$= I(x)y' + I(x)P(x)y = (I(x)y)' = I'(x)y + I(x)y'$$

$$I(x)P(x)y = I'(x)y$$

$$I(x)P(x) = I'(x)$$

$u = I(x)$
$du = I'(x)dx$

$$\int P(x)\,dx = \int \frac{I'(x)}{I(x)}\,dx = \int \frac{du}{u} = \ln|u| = \ln|I(x)|$$

Assume $I(x) \geq$

$$e^{\int P(x)\,dx} = e^{\ln(I(x))} \Rightarrow I(x) = e^{\int P(x)\,dx}$$

Let $I(x) = e^{\int P(x)\,dx}$ is the integrating factor for $y' + P(x)y = Q(x)$

(5) $y' + 2y = 2e^x$

$P(x) = 2$

$e^{\int 2dx} = e^{2x+C}$

Ditch it.

$e^{2x}[y' + 2y] = e^{2x} \cdot 2e^x$

$e^{2x}[y' + 2y]$
$= e^{2x}y' + 2e^{2x}y$
$e^{2x} \cdot y]$

$\frac{d}{dx}[e^{2x}y] = 2e^{3x}$

$\int d[e^{2x}y] = \int 2e^{3x}dx$

$y' + P(x)y = Q(x)$

1st-ORDER LINEAR ODE.

$e^{2x}y = \frac{2}{3}e^{3x} + C$

$y = \frac{2}{3}e^x + Ce^{-2x}$

Check: $y' + 2y$

$= \frac{2}{3}e^x - 2Ce^{-2x} + 2(\frac{2}{3}e^x + Ce^{-2x})$

$= \frac{2}{3}e^x - 2Ce^{-2x} + \frac{4}{3}e^x + 2Ce^{-2x}$

$= 2e^x$ ✓ Cool.

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(11) $\sin x\, y' + \cos x\, y = \sin(x^2)$ Pg 639

Formula $\boxed{4}$

$y' + P(x)y = Q(x)$

$\sin x\, y' + \cos x\, y = (\sin x\, y)' = \sin(x^2)$

$\sin x\, y = \int \sin(x^2)\,dx$

$u = x^2$ $du = 2x\,dx$
leads NOwhere.

$$\boxed{y = \frac{\int \sin(x^2)\,dx}{\sin x}} = \int$$

$y' + \cot x\, y = \frac{\sin(x^2)}{\sin x}$

$e^{\int \cot x\,dx} = e^{\ln|\sin x|} = |\sin x|$

$\int \frac{\cos(x)}{\sin x}$ $= \sin x$ w/ appropriate restrictions.

Goal: To write LHS as the derivative of $I(x)y$

Eq'n $y' + P(x)y = Q(x)$

$$\underbrace{I(x)y' + I(x)P(x)y}_{(I(x)y)'} = I(x)Q(x)$$

$$(I(x)y)' = I(x)y' + I(x)P(x)y$$

$$I'(x)y + I(x)y' = I(x)y' + I(x)P(x)y$$

$$I'(x)y = I(x)P(x)y$$

$$I'(x) = I(x)P(x)$$

$$\frac{dI}{dx} = IP$$

$$\int \frac{dI}{I} = \int P\,dx$$

$$e^{\ln(I(x))} = e^{\int P\,dx}$$

$$I(x) = e^{\int P(x)\,dx}$$