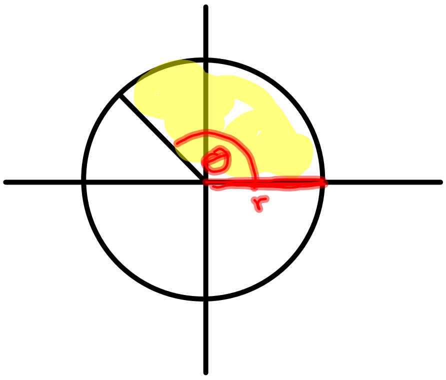


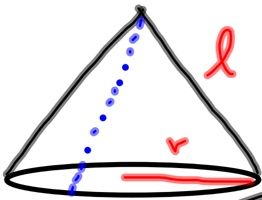
πr^2

$= \frac{1}{2} \cdot 2\pi r^2$ Area of circle

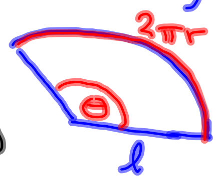
$= \frac{1}{2} r^2 \cdot 2\pi \rightsquigarrow \theta = 2\pi$

$A = \frac{1}{2} r^2 \theta$, where θ is in radians

Surface area of cone (sides only)



cut & lay flat



looks like a sector of a circle of radius l.

$\boxed{\text{§8.3 \#35}}$

Area $= \frac{1}{2} l^2 \theta$

Recall arc length is radius · angle

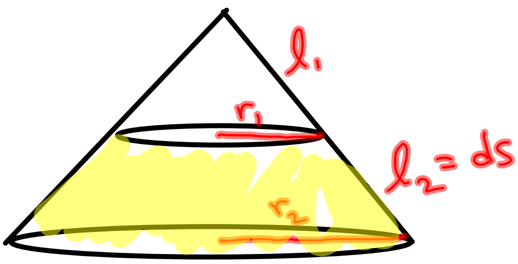
$= l \cdot \theta = 2\pi r$

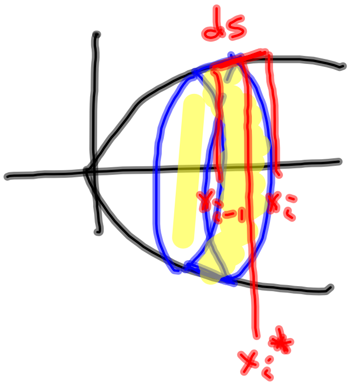
$\Rightarrow \theta = \frac{2\pi r}{l}$

$\Rightarrow$ Area of cone is

$\frac{1}{2} l^2 \theta = \frac{1}{2} l^2 \cdot \frac{2\pi r}{l} = \pi r l$

Area of a frustum





Area = Area of BIG Cone − Area of Small Cone

$= \pi r_2 (l_1 + l_2) - \pi r_1 l_1$

$= \pi (r_2 l_1 + r_2 l_2 - r_1 l_1)$

$= \pi (r_1 l_2 + r_2 l_2)$

$= \pi (r_1 + r_2) l_2$

$= 2\pi \left(\frac{r_1 + r_2}{2} \right) l_2$

Similar Triangles

$\frac{l_1}{r_1} = \frac{l_1 + l_2}{r_2}$

$r_2 l_1 = r_1 l_1 + r_1 l_2$

$\Rightarrow r_2 l_1 - r_1 l_1 = r_1 l_2$

The i^{th} Frustum:

$= 2\pi \left(\frac{f(x_{i-1}) + f(x_i)}{2} \right) ds$

$\approx 2\pi \left(\frac{f(x_i^*) + f(x_i^*)}{2} \right) ds$

$= 2\pi \; f(x_i^*)\, ds$, where x_i^* is between x_{i-1} & x_i

as $dx \longrightarrow$ small,

$f(x_{i-1}) \approx f(x_i^*) \approx f(x_i)$

ds = arc length of i^{th} interval

$$\sqrt{1+(f'(x_i))^2}\,dx$$

So surface area of the solid obtained by rotating f(x) about the x-axis is

$$\approx 2\pi \sum_{k=1}^{n} f(x_k^*)\sqrt{1+f'(x_k^*)^2}\,dx$$

$$\xrightarrow{n\to\infty} 2\pi\int_a^b f(x)\sqrt{1+f'(x)^2}\,dx$$

$$= 2\pi\int y\,ds$$ revolving about x-axis.

Revolving About the y-axis

$$2\pi\int_c^d g(y)\sqrt{1+(g'(y))^2}\,dy$$

$$= 2\pi\int x\,ds$$ revolving about y-axis

Set up:

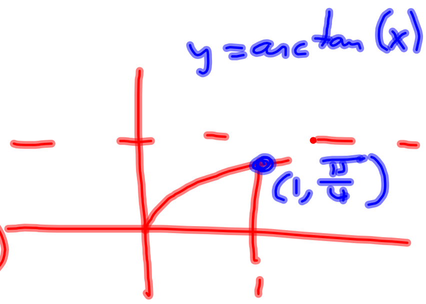
$f'(x) = \frac{1}{x^2+1}$

$y = \arctan(x)$, $0 \le x \le 1$

(a) about x-axis (b) about y-axis

(a) $2\pi \int y \, ds = 2\pi \int_0^1 \arctan(x) \sqrt{1 + \left(\frac{1}{x^2+1}\right)^2} \, dx$

(b) $2\pi \int x \, ds = 2\pi \int_0^1 x \sqrt{1 + \left(\frac{1}{x^2+1}\right)^2} \, dx$



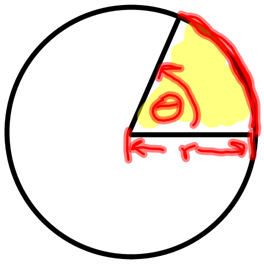
$y=1$

Blunt Force:

$y = \arctan(x) \Longrightarrow x = \tan(y) = g(y)$

$g'(y) = \sec^2(y)$

(b) $2\pi \int x \, ds = 2\pi \int_0^{\frac{\pi}{4}} \tan(y) \sqrt{1 + (\sec^2(y))^2} \, dy$



$\left.\begin{array}{l}\text{Area} = \frac{1}{2}r^2\theta \\ \text{Arc Length} = r\theta\end{array}\right\}$ ok?

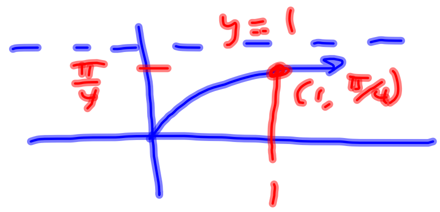
$y = \arctan(x) \quad 0 \le x \le 1$

Area of surface of revolution when revolving around y-axis

Recall:

(b) $2\pi \int x \, ds = 2\pi \int_0^1 x \sqrt{1 + \left(\frac{1}{x^2+1}\right)^2} \, dx$

without this neat way:



$y = \arctan(x) \Longrightarrow$

$x = \tan(y) = g(y)$

$\frac{dx}{dy} = \sec^2 y$

This gives $2\pi \int_0^{\frac{\pi}{4}} \tan(y) \sqrt{1 + \sec^4(y)} \, dy$

So you <u>Can</u> find $x = f^{-1}(y)$, but it's not necessary AND it relies on y being 1-to-1 on the region in question

ρ = density of liquid

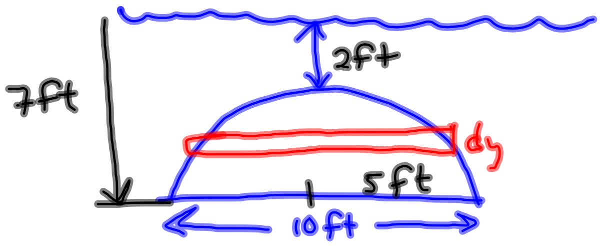
g = acceleration of gravity

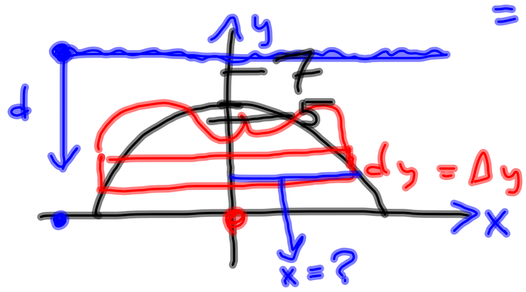
$F = \text{Force} = ma = \rho \cdot V \cdot g = \rho \cdot g \cdot \underset{\text{rect}}{A}\,\underset{\text{depth}}{d}$

Mass density of water in American is

62.5 lbs/ft^2

Find Force on this semicircle:





$\rho g A d$

$= \rho g \cdot 2\sqrt{25-y^2}\,\Delta y\,(7-y)$

$x^2 + y^2 = 5^2$

$x^2 = 25 - y^2$

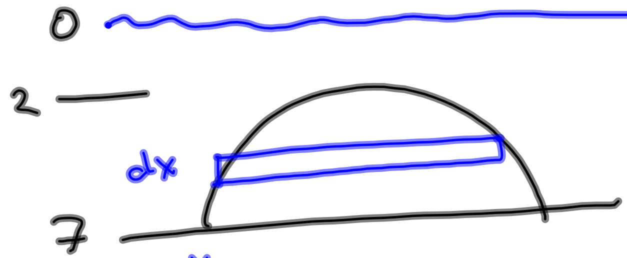
$x = \pm\sqrt{25-y^2}$

Take the positive

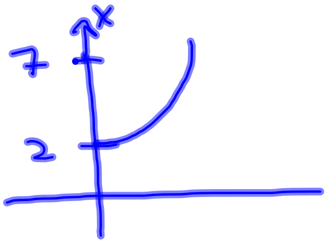
$$F \approx \sum_{k=1}^{n} 2\rho g (7-y)\sqrt{25-y^2}\,\Delta y$$

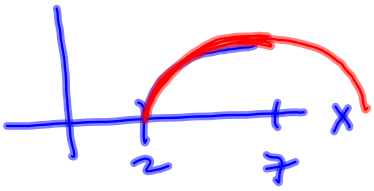
$$\xrightarrow{n \to \infty} 2\rho g \int_0^5 (7-y)\sqrt{25-y^2}\,dy$$

$$= 125 \int_0^5 (7-y)\sqrt{25-y^2}\,dy$$



$$2\rho g A d$$
$$= 2\rho g \, d A$$
$$= 2\rho g x$$





$$(x-7)^2 + y^2 = 5^2$$
$$\sqrt{y^2} = \sqrt{25-(x-7)^2}$$
$$y = \sqrt{25-(x-7)^2}$$

$$125 \int_2^7 x \sqrt{25-(x-7)^2} \, dx$$