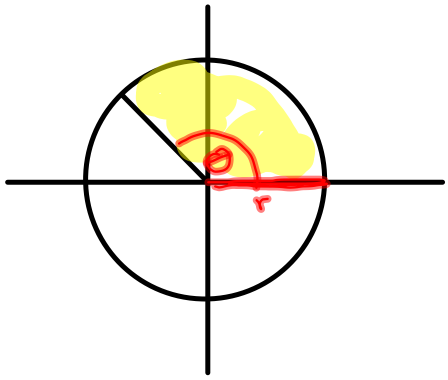


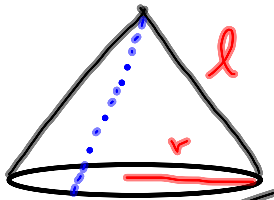
πr^2

$= \frac{1}{2} \cdot 2\pi r^2$ Area of circle

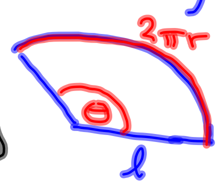
$= \frac{1}{2} r^2 \cdot 2\pi \rightarrow \theta = 2\pi$

$A = \frac{1}{2} r^2 \theta$, where θ is in radians

Surface area of cone (sides only)



cut & lay flat



looks like a sector of a circle of radius ℓ.

$\boxed{\S 8.3 \ \#35}$

Area $= \frac{1}{2} \ell^2 \theta$

Recall arc length is radius · angle

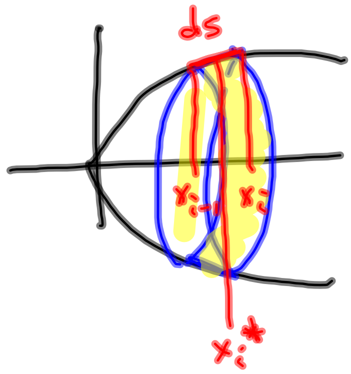
$= \ell \cdot \theta = 2\pi r$

$\Rightarrow \theta = \frac{2\pi r}{\ell}$

$\Rightarrow$ Area of cone is

$\frac{1}{2} \ell^2 \theta = \frac{1}{2} \ell^2 \cdot \frac{2\pi r}{\ell} = \pi r \ell$

Area of a frustum



Area = Area of BIG Cone − Area of Small Cone

$= \pi r_2(l_1+l_2) - \pi r_1 l_1$

$= \pi(r_2 l_1 + r_2 l_2 - r_1 l_1)$

$= \pi(r_1 l_2 + r_2 l_2)$

$= \pi(r_1+r_2) l_2$

$= 2\pi\left(\frac{r_1+r_2}{2}\right) l_2$

Similar Triangles

$\frac{l_1}{r_1} = \frac{l_1+l_2}{r_2}$

$r_2 l_1 = r_1 l_1 + r_1 l_2$

$\Rightarrow r_2 l_1 - r_1 l_1 = r_1 l_2$

The i^{th} Frustum:

$= 2\pi\left(\frac{f(x_{i-1}) + f(x_i)}{2}\right) ds$

as $dx \to$ small,

$f(x_{i-1}) \approx f(x_i^*) \approx f(x_i)$

$\approx 2\pi\left(\frac{f(x_i^*) + f(x_i^*)}{2}\right) ds$

$= 2\pi \, f(x_i^*) \, ds$, where x_i^* is between x_i & x_{i-1}.

ds = arc length of i^{th} interval

$$\sqrt{1+(f'(x_k))^2}\,dx$$

So surface area of the solid obtained by rotating $f(x)$ about the x-axis is

$$\approx 2\pi \sum_{k=1}^{n} f(x_k^*)\sqrt{1+f'(x_k^*)^2}\,dx$$

$$\xrightarrow{n\to\infty} 2\pi \int_a^b f(x)\sqrt{1+f'(x)^2}\,dx$$

$= 2\pi \int y\,ds$ revolving about x-axis.

Revolving About the y-axis

$$2\pi \int_c^d g(y)\sqrt{1+(g'(y))^2}\,dy$$

$= 2\pi \int x\,ds$ revolving about y-axis

Oct 5-12:44 PM

Set up:

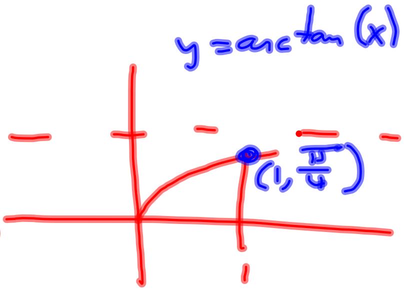
$y = \arctan(x)$, $0 \le x \le 1$

$f'(x) = \frac{1}{x^2+1}$

(a) about x-axis (b) about y-axis

(a) $2\pi \int y \, ds = 2\pi \int_0^1 \arctan(x) \sqrt{1 + \left(\frac{1}{x^2+1}\right)^2} \, dx$

(b) $2\pi \int x \, ds = 2\pi \int_0^1 x \sqrt{1 + \left(\frac{1}{x^2+1}\right)^2} \, dx$



Blunt Force:

$y = \arctan(x) \Longrightarrow x = \tan(y) = g(y)$

$g'(y) = \sec^2(y)$

(b) $2\pi \int x \, ds = 2\pi \int_0^{\frac{\pi}{4}} \tan(y) \sqrt{1 + (\sec^2(y))^2} \, dy$