$$\int_0^4 \left(x^5\sqrt{\cosh(x)}\right)dx$$

$n = 8$ rectangles

$$\frac{b-a}{n} = \frac{4}{8} = \frac{1}{2}$$

$x_0 = 0$

$x_1 = \frac{1}{2}$

$x_2 = 1$

$x_3 = \frac{3}{2}$

$\bar{x}_1 = \frac{1}{4}$

$\bar{x}_2 = \frac{3}{4}$

$\bar{x}_3 = \frac{5}{4}$

$\bar{x}_4 = \frac{7}{4}$

$\vdots$

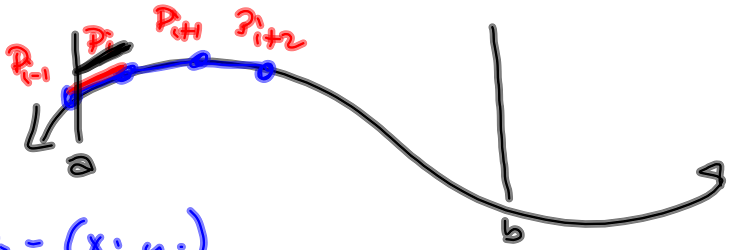
$$S_M = \Delta x \left[\sum_{k=1}^{8} f(\bar{x}_k)\right]$$

≈ 11012.10182

```
        2799.930657
Y1(1/4)+Y1(3/4)+
Y1(5/4)+Y1(7/4)+
Y1(9/4)+Y1(11/4)
+Y1(13/4)+Y1(15/
4)
        22024.20363
```

```
Y1(5/4)+Y1(7/4)+
Y1(9/4)+Y1(11/4)
+Y1(13/4)+Y1(15/
4)
        22024.20363
Ans/2
        11012.10182
```

§9.1 Arc Length



$P_i = (x_i, y_i)$

$P_{i-1} = (x_{i-1}, y_{i-1})$

$$|P_{i-1}P_i| = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}$$
$$= \sqrt{\Delta x_i^2 + \Delta y_i^2}$$

Scratch:

$\Delta y_i = f(x_i) - f(x_{i-1})$ If f is smooth, then $\exists\ x_i^* \in (x_{i-1}, x_i)$ such that $\boxed{\text{MVT}}$

$$f(x_i) - f(x_{i-1}) = f'(x_i^*)(x_i - x_{i-1})$$

Continuing: $= f'(x_i^*)\,\Delta x_i$

$$\sqrt{\Delta x_i^2 + f'(x_i^*)^2\,\Delta x_i^2} = \Delta x_i \sqrt{1 + f'(x_i^*)^2}$$

So length of the i^{th} segment is approximately

$$\sqrt{1 + f'(x_i^*)^2}\ \Delta x_i$$

Oct 4-12:22 PM

So length of the $i^{\underline{th}}$ segment is approximately

$$\sqrt{1+f'(x_i^*)^2}\ \Delta x_i$$

And the length of the arc is approximately

$$S \approx \sum_{k=1}^{n} \sqrt{1+f'(x_k^*)^2}\ \Delta x$$

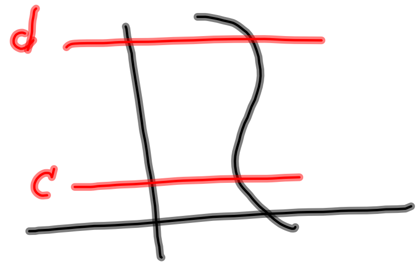
Arc Length Integral

$$\xrightarrow{n \to \infty} \boxed{\int_a^b \sqrt{1+(f'(x))^2}\ dx}$$

Formula $\boxed{2}$ §9.1

Liebniz : $\boxed{3}$ $\displaystyle\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\ dx$

$\boxed{4}$ If $x = g(y)$



$$S = \int_c^d \sqrt{1 + g'(y)^2}\, dy$$

$$= \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Arc Length as an increasing function:

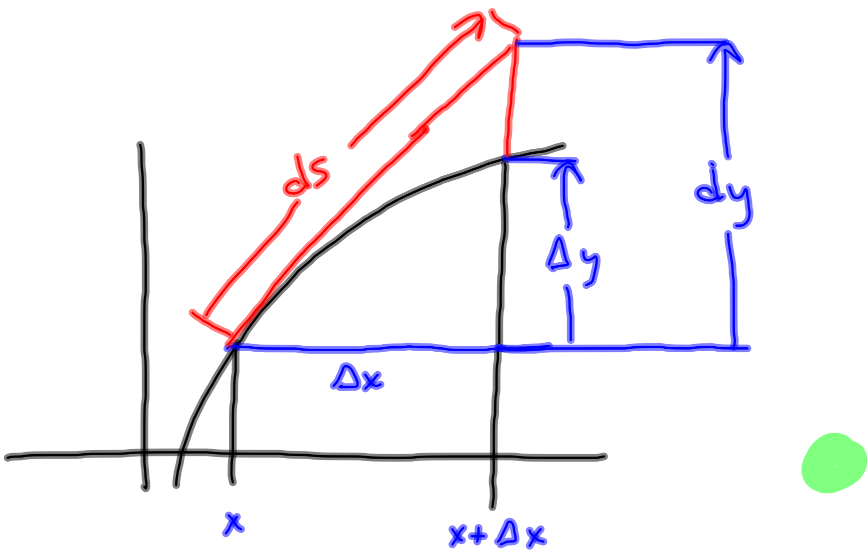
$$S(x) = \int_a^x \sqrt{1 + (f'(t))^2}\, dt$$

(s) s

$$S(x) = \int_a^x \sqrt{1 + (f'(t))^2}\, dt$$

$$\frac{dS}{dx} = \sqrt{1 + f'(x)^2} \quad \text{, so}$$

$dS = \sqrt{1 + f'(x)^2}\, dx$ is a way to approximate Δs = change in arc length correspon-ding to a small change $\Delta x \cong dx$ in x.



Let $y = 1 + 6x^{\frac{3}{2}}$ Find arc length for $0 \le x \le 1$.

$(ab)^2 = a^2 b^2$

C9 Homework will be posted this afternoon

$y' = f'(x) = \left(9x^{\frac{1}{2}}\right)$

$$\int_0^1 \sqrt{1 + (f'(x))^2}\, dx = \int_0^1 \sqrt{1 + 81x}\, dx$$

$u = 81x + 1$ $du = 81dx$

$(x^{\frac{1}{2}})^2 = x$

$x = 0$ $u = 1$

$x = 1$ $u = 82$

$$= \frac{1}{81}\int_1^{82} \sqrt{u}\, du = \frac{1}{81}\left[\frac{2u}{3}^{\frac{3}{2}}\right]_1^{82}$$

$$= \cdots = \frac{2}{243}\left[82\sqrt{82} - 1\right]$$