$f(x) = x \sin x$ on $[0,4]$

we need $K \geq \max_{[0,4]} \{ |f''(x)| \}$

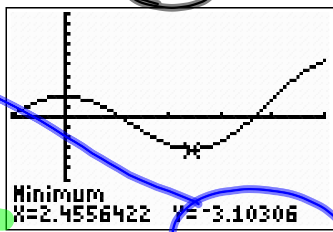
$f'(x) = \sin x + x\cos x$

$f''(x) = \cos x + \cos x - x \sin x$

$= 2\cos x - x \sin x$

Let $g(x) = 2\cos x - x \sin x$ Find its max on $[0,4]$ in absolute value.

Let $K = 4$, from graph.



$K=3.2$

$g'(x) = -2\sin x - \sin x - x\cos x$

$= -3\sin x - x\cos x \overset{\text{SET}}{=} 0$

$-3\sin x = x \cos x$ or just use a grapher.

Too hard.

An analyst would do this:

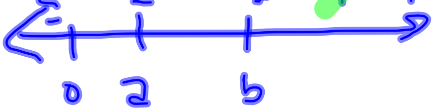
$|f''(x)| = |2\cos x - x \sin x|$

$\leq |2\cos x| + |x \sin x|$

$x \in [0,4]$

$\leq 2 \cdot 1 + |x||\sin x|$

$\leq 2 + 4 \cdot 1 = 6$



Adding vectors

Triangle Inequality

$|a+b| \leq |a| + |b|$

$a+b = |a+b| = |a| + |b|$ if $a, b \geq 0$

Quick & Dirty

$K = 6 \geq |f''(x)|$ on $[0,4]$

$|3+2| = |5| = |3| + |2|$

$|3 + -2| = 1 < |3| + |-2| = 5$

$|a+b| \leq |a| + |b|$ always.

Matt noticed in Take-Home Test 2, we had $\int_0^4 \sqrt{x}\sin x\,dx$.

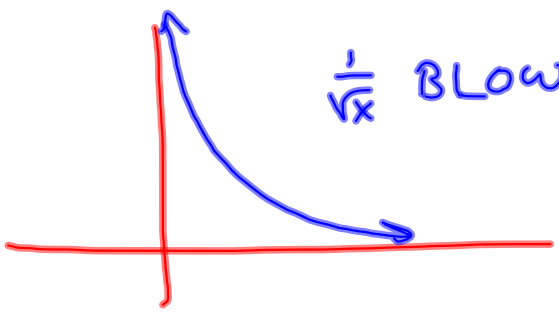
So I changed it to $x\sin x$ for parts c&d.

Our estimates for K break down, here, because...

$$f'(x) = \frac{1}{2\sqrt{x}}\sin x + \sqrt{x}\cos x \text{ is}$$

not bounded on $[0,4]$.

f'' same problem, so K is impossible to find for your error estimate.



$\frac{1}{\sqrt{x}}$ BLOWS UP @ $x=0$

$x > 5$ $\quad \frac{1}{x} < \frac{1}{x-1}$

Assume $x >$ pretty big

$$\frac{1}{x^2-5x-6} > \frac{1}{x^2}$$

$$\frac{1}{x^2+5x+7} < \frac{1}{x^2}$$

$\int_1^\infty \frac{1}{x^2}dx$ converges

x^2-6x+3^2-9-5

$(x-3)^2-14$

$\int_1^\infty \frac{1}{x^2+5x+6}dx$ converges because the 1^{st} one does.

$\int_1^\infty \frac{1}{x}dx \nexists$ $\quad \boxed{\text{Comparison}}$ Method.

$u = x-3$

$$\int_{50}^\infty \frac{dx}{x^2-6x-5} = \int_{50}^\infty \frac{dx}{(x-3)^2-14} = \int_{47}^\infty \frac{du}{u^2-14},$$

It will converge, by "semi-comparison" to $\int \frac{dx}{x^2}$

Oct 3-12:35 PM

$$\int_1^\infty \frac{dx}{x^2} \geq \int_1^\infty \frac{dx}{e^x}$$ does converge.

What about $\int_1^\infty e^{-2x}\,dx \leq \int_1^\infty e^{-x}\,dx$

$\frac{dx}{e^{2x}}$ $\frac{dx}{e^x}$

So it converges, by comparison.

$e^{2x} > e^x \quad \forall x > 0$

$\frac{1}{e^{2x}} < \frac{1}{e^x} \quad \forall x > 0$

Oct 3-12:45 PM

Let $u^2 = \ldots$

Let $u = \ldots$ is better.

Starting with this always gives $\sqrt{u^2}$ at some point & then you have to argue why

$$\sqrt{u^2} = u$$

$\sqrt{u^2} = |u|$ & you need a rationale for dropping the absolute values.

#77 8.5 II

$$\int \frac{\sqrt{x}\,dx}{1+x^3}$$

~~$u=\sqrt{x}$ is one way~~

$u = x^3 + 1$

$u-1 = x^3$

$(u-1)^{\frac{1}{3}} = x$

$\frac{1}{3}(u-1)^{-\frac{2}{3}}du = dx$

$\sqrt{x} = (u-1)^{\frac{1}{6}}$

$$= \int \frac{(u-1)^{\frac{1}{6}}\left(\frac{1}{3}(u-1)^{-\frac{2}{3}}\right)du}{u}$$

$$= \frac{1}{3}\int \frac{\left(\sqrt{u-1}\right)^{-1}}{u}\,du$$

owie!

Let $u = \sqrt{x}$

$u^2 = x$

Kelly said

$u = \sqrt{x^3}$

$$\int \frac{2u^2}{1+u^6}\,du$$

Let $v = u^{\frac{3}{2}}$

$u = v^{\frac{2}{3}}$

$$= 2\int \frac{v^{\frac{4}{3}}}{1+v^4}\cdot\frac{2}{3v^{\frac{1}{3}}}\,dv = \frac{4}{3}\int \frac{v}{1+v^4}\,dv$$