$\int_0^1 \frac{dx}{x}$ $\frac{1}{x} \nexists$ @ $x=0$ and the integral doesn't exist. → converge

$\int_0^1 \frac{dx}{\sqrt{x}}$ DOES exist! p-test

$\int_0^1 x^{-\frac{1}{2}}dx = \int_0^1 \frac{dx}{x^{\frac{1}{2}}}$ $p = \frac{1}{2}$ $0 < p < 1$ $\nexists$

$\int_0^x$ or $\int_{-1}^2$ $\exists$ → converge,

$$\boxed{\int_{.1}^1 \frac{dx}{x^{1/2}} \text{ or } \int_{.0001}^1 \frac{dx}{x^{1/2}} \text{ No PROB.}}$$

Because FTC relates to CONTINUOUS INTEGRANDS.

Check it out

$$\lim_{t\to 0^+} \int_t^1 \frac{dx}{x^{1/2}} = \lim_{t\to 0^+} \left[2x^{\frac{1}{2}}\right]_t^1 = 2(1)^{\frac{1}{2}} - \lim_{t\to 0^+}\left[2t^{1/2}\right] = 2$$

$\int_0^1 \frac{dx}{x^{3/4}}$ $\int_0^1 \frac{dx}{x^{3/2}}$ $\int_0^1 \frac{dx}{x^{.999}}$

Sep 30-12:17 PM

$$\int_1^{\infty} \frac{dx}{x}$$

p-test

Now, we need $p > 1$ for infinite intervals.

$$\int_1^{\infty} \frac{dx}{x^{1/2}} = 2x^{\frac{1}{2}}\Big]_1^{\infty}$$

$$\int_1^{\infty} \frac{dx}{x^2} = \lim_{t\to\infty} \int_1^{t} \frac{dx}{x^2} = \lim_{t\to\infty} \Big[-x^{-1}\Big]_1^{t}$$

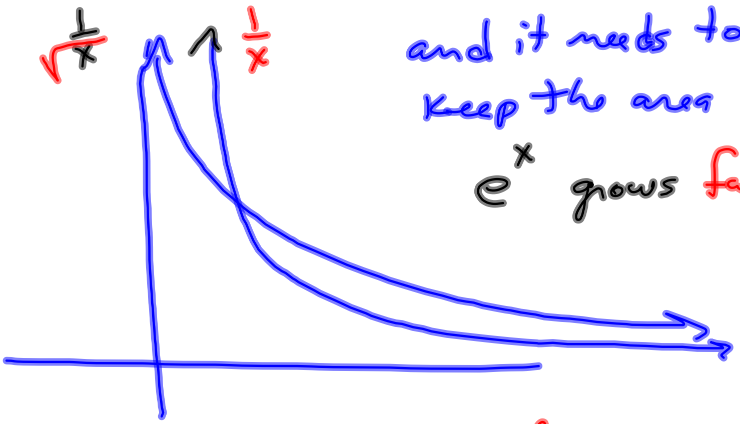
$$= \lim_{t\to\infty} \left[-\frac{1}{t} - \left(-\frac{1}{1}\right)\right] = 1$$

Sep 30-12:28 PM

$\int_1^\infty \frac{dx}{x^p}$ converges if $p > 1$, because x^p grows faster than x.

So $\frac{1}{x^p}$ shrinks faster than $\frac{1}{x}$

and it needs to shrink to keep the area finite



e^x grows faster than x

$\int_1^\infty \frac{dx}{e^x}$ converges

$\ln(x)$ grows slower than x

(e,e) (e^2,e^2) (e^3,e^3) (e^4,e^4)

$y=x$ $(1,1)$, $(10,10)$, $(100,100)$, $(1000,1000)$

$\log x = y$ $(1,0)$, $(10,1)$, $(100,2)$, $(1000,3)$

$\ln x = y$ $(1,0)$, $(e,1)$, $(e^2,2)$, $(e^3,3)$

11. $\int_{-\infty}^{\infty} \frac{x}{1+x^2}\,dx = 0$?

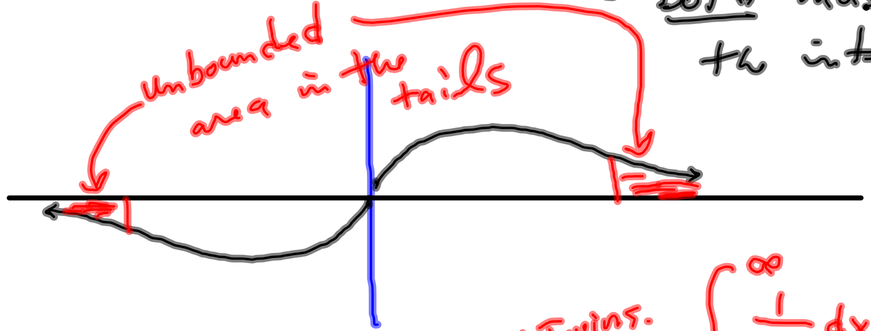
$$\int_{-2000000}^{2000000} \frac{x}{1+x^2}\,dx = 0$$

But

$$\int_{-\infty}^{\infty} \frac{x}{1+x^2}\,dx = \int_{-\infty}^{0} \frac{x}{1+x^2}\,dx + \int_{0}^{\infty} \frac{x}{1+x^2}\,dx$$

But NEITHER of these converges!

AND BOTH must converge for the integral to exist.



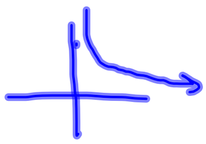
Evil Twins.

$\int_1^{\infty} \frac{1}{x}\,dx$ has same issue.

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges

$\int_1^{\infty} \frac{1}{x^2}\,dx$ converges



$\sum_{n=k}^{\infty} \frac{1}{n}$ is the "k-tail"

$$\int_k^{\infty} \frac{1}{x}\,dx$$

$$\int_{-\infty}^{\infty} x^3 e^{-x^4}\,dx = -\frac{1}{4}\int_{-\infty}^{\infty} e^{-x^4}\cdot(-4x^3\,dx)$$

is a weird one. Problems @ $x=0$ and we're running to ∞. → None

Let's look at this piece:

$$-\frac{1}{4}\int_{1}^{\infty} e^{-x^4}(-4x^3\,dx)$$

Let $u=-x^4$. Then $du=-4x^3\,dx$

$x=1 \longrightarrow u=-1$

$x\to\infty \Longrightarrow -x^4 \longrightarrow -\infty$

$$-\frac{1}{4}\int_{-1}^{-\infty} e^{u}\,du = -\frac{1}{4}\lim_{t\to-\infty}\int_{-1}^{t} e^{u}\,du$$

$$= -\frac{1}{4}\lim_{t\to-\infty}\left[e^{u}\right]_{-1}^{t} = -\frac{1}{4}\lim_{t\to-\infty}\left[e^{t}-e^{-1}\right]$$

$$= -\frac{1}{4}(-e^{-1}) = +\frac{1}{4e}$$

Exponentials Dominate Power functions when $x \longrightarrow \pm$ BIG.

$$\int_{-\infty}^{\infty} x^3 e^{-x^4}\,dx = \lim_{t\to-\infty}\int_t^0 x^3 e^{-x^4}\,dx + \lim_{t\to\infty}\int_0^t x^3 e^{-x^4}\,dx$$

Finish Monday

TEST 2 By Wednesday, End-of-day