8.5#57

$$\int x\sqrt[3]{x+c}\,dx \qquad u=x \quad du=dx$$
$$dv=(x+c)^{1/3}dx \longrightarrow$$
$$v=\frac{3}{4}(x+c)^{\frac{4}{3}}$$

$$uv-\int v\,du = x\cdot\frac{3}{4}(x+c)^{\frac{4}{3}}-\frac{3}{4}\int(x+c)^{\frac{4}{3}}dx$$

$u=x+c$
$du=dx$

$$=\frac{3}{4}x(x+c)^{\frac{4}{3}}-\frac{3}{4}\cdot\frac{3}{7}(x+c)^{\frac{7}{3}}+K \overset{?}{=} \frac{3}{7}(x+c)^{\frac{7}{3}}-\frac{3}{4}c\,(x+c)^{\frac{4}{3}}+K$$

Evil Student

Book

$$\frac{3}{4}(x+c)^{\frac{4}{3}}\left[x-\frac{3}{7}(x+c)\right]=\frac{3}{4}(x+c)^{\frac{4}{3}}\left[\frac{4}{7}x-\frac{3}{7}c\right]$$

Leads nowhere.

$$=\frac{3}{7}x(x+c)^{\frac{4}{3}}-\frac{9}{28}c\,(x+c)^{\frac{4}{3}}$$

still not book.

Hmmmmm

$$\frac{3}{7}x(x+c)^{\frac{4}{3}} - \frac{9}{28}c(x+c)^{\frac{4}{3}}$$

$$= \frac{3}{7}(x+c-c)(x+c)^{\frac{4}{3}} - \frac{9}{28}c(x+c)^{\frac{4}{3}}$$

$$= \frac{3}{7}\left[(x+c)^{\frac{7}{3}} - c(x+c)^{\frac{4}{3}}\right] - \frac{9}{28}c(x+c)^{\frac{4}{3}}$$

$$= \frac{3}{7}(x+c)^{\frac{7}{3}} - \frac{3}{7}c(x+c)^{\frac{4}{3}} - \frac{9}{28}c(x+c)^{\frac{4}{3}}$$

$$= \frac{3}{7}(x+c)^{\frac{7}{3}} - \frac{3}{4}c(x+c)^{\frac{4}{3}} \quad = \text{Book}$$

Find lower bound on $n \ni \int_0^{\frac{8\pi}{3}} \sin(4x)\,dx$

(i) $|E_T| < .0001$

(ii) $|E_M| < .0001$

(iii) $|E_S| < .0001$

E_M & E_T both use f'' in the estimate

(i) $n > 2800$ as lower bound on n.

(ii) $|E_M| \leq \frac{K(b-a)^3}{24n^2} = \frac{1}{2}|E_T| = \frac{1}{2}\,\frac{K(b-a)^3}{12n^2}$

We found yesterday that $|f''(x)| \leq 16$ on $[0, \frac{8\pi}{3}]$

$$|E_M| \leq \frac{16\left(\frac{8\pi}{3}\right)^3}{24n^2} < .0001$$

$$\frac{16\left(\frac{8\pi}{3}\right)^3}{.0024} < n^2$$

$$n > \sqrt{\frac{16\left(\frac{8\pi}{3}\right)^3}{.0024}} \quad \text{etc.}$$

$|E_S|$ will involve

$$|f^{(4)}(x)| \leq 256 = K$$

$$K \geq |256\sin(4x)| = 256\,|\sin(4x)| \leq 256 \cdot 1 = 256$$

$f(x) = \sin(4x)$

$f^{(1)}(x) = 4\cos(4x)$

$f^{(2)}(x) = -16\sin(4x)$

$f^{(3)}(x) = -64\cos(4x)$

$f^{(4)}(x) = 256\sin(4x)$

Estimate $\int_0^{\pi} x^2 \sin(x)\,dx$ using $n=4$ intervals.

(i) Trapezoidal Rule

$$T_4 = \frac{\Delta x}{2}\left[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+f(x_4)\right]$$

$$\Delta x = \frac{\pi-0}{4} = \frac{\pi}{4}$$

$a = x_0 = 0$

$x_1 = \frac{\pi}{4}$

$x_2 = \frac{\pi}{2}$

$x_3 = \frac{3\pi}{4}$

$x_4 = \pi$

$$T_4 = \frac{\frac{\pi}{4}}{2}\left[0^2\cdot\sin(0)+2\left(\frac{\pi}{4}\right)^2\sin\left(\frac{\pi}{4}\right) + 2\left(\frac{\pi}{2}\right)^2\sin\left(\frac{\pi}{2}\right)+2\left(\frac{3\pi}{4}\right)^2\sin\left(\frac{3\pi}{4}\right)+ \pi^2\sin(\pi)\right] = \frac{\pi}{8}\left[\frac{\pi^2}{8}\cdot\frac{\sqrt2}{2} + \frac{\pi^2}{2}\cdot 1 + \frac{9\pi^2}{8}\cdot\frac{\sqrt2}{2}\right] = \frac{\pi^3}{8}\left[\frac{\sqrt2}{16} + \quad + \frac{9\sqrt2}{16}\right]$$

$$= \frac{\pi^3}{8}\left[\frac{10\sqrt2}{16} + \frac{8}{16}\right] = \frac{\pi^3}{8\cdot 16}\left[10\sqrt2+8\right]$$

$$\approx 5.363634246$$

$\boxed{\text{8.7 Monday}}$