§8.5 #37

$$\int_0^{\frac{\pi}{4}} \cos^2\theta \tan^2\theta \, d\theta = \int_0^{\frac{\pi}{4}} \sin^2\theta \, d\theta = \frac{1}{2}\int_0^{\frac{\pi}{4}} (1-\cos(2\theta))\,d\theta$$

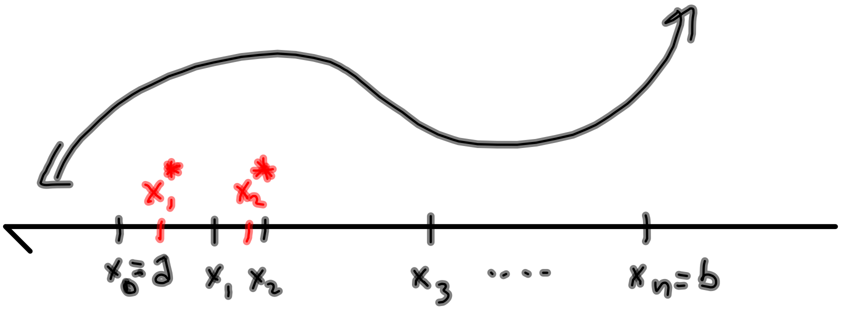
$$= \frac{1}{2}\left[\theta - \frac{1}{2}\sin(2\theta)\right]_0^{\frac{\pi}{4}} = \frac{1}{2}\left[\left(\frac{\pi}{4} - \frac{1}{2}\sin\left(\frac{\pi}{2}\right)\right) - \left(0 - \frac{1}{2}\sin(0)\right)\right]$$

$$= \frac{\pi}{8} - \frac{1}{4}$$

Section 8.7 Approximate Integration

Recall: f cnt^s on $[a,b]$, $f(x) \geq 0$. Then area between $f(x)$ and x-axis is approximately

$$A \approx \sum_{k=1}^{n} f(x_k^*)\Delta x_k, \quad x_k^* \in [x_{k-1}, x_k]$$



From CALC I, using $x_k^* = x_k$ = Right Endpoint.

$\Delta x_k = \frac{b-a}{n}$, so intervals are equal width.

So Area $\approx \sum_{k=1}^{n} f(x_k^*)\Delta x_k = \sum_{k=1}^{n} f(x_k)\cdot \frac{b-a}{n}$

$x_1 = x_0 + \Delta x = a + \frac{b-a}{n}$

$x_2 = x_1 + \Delta x = \cdots = a + 2\left(\frac{b-a}{n}\right)$

$\vdots$

$x_k = a + k\left(\frac{b-a}{n}\right) = a + \frac{b-a}{n}k$

$$= \sum_{k=1}^{n}\left(f\left(a + \frac{b-a}{n}k\right)\frac{b-a}{n}\right)$$

$$= \frac{b-a}{n}\sum_{k=1}^{n} f\left(a + \frac{b-a}{n}k\right)$$

NOTE $n \to \infty$ insures that $\Delta x_k \to 0$, provided the intervals are equal width.

But if they are NOT equal width, then $n \to \infty$ isn't strong enough to give us

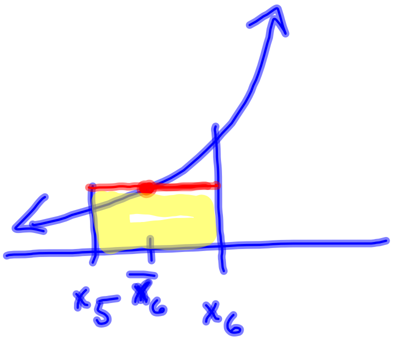
THEY ALL have to get skinny.

$\xrightarrow{n \to \infty} \int_a^b f(x)\,dx = \text{AREA}$

Midpoint Rule: $x_k^* = \dfrac{x_{k-1}+x_k}{2} = \bar{x}_k$

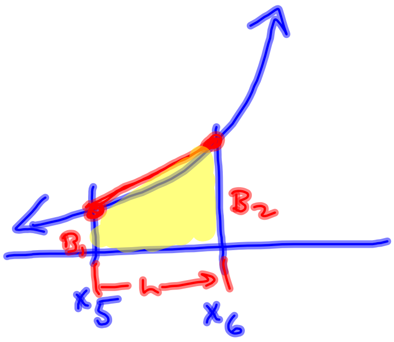
$$M_n = \sum_{k=1}^{n} f(\bar{x}_k)\,\Delta x_k$$

Equal width version: $\dfrac{b-a}{n}\displaystyle\sum_{k=1}^{n} f(\bar{x}_k)$



Area 6

TRAPEZOIDAL RULE



Area 6

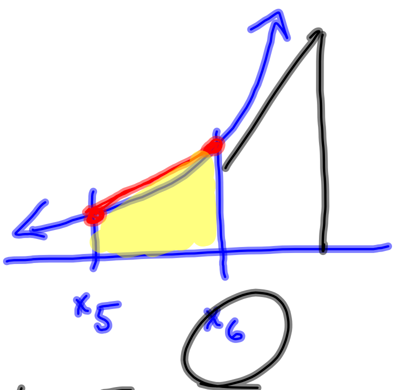
Area of Trapezoid is

$\frac{1}{2}(B_5+B_6)h$

$= \frac{1}{2}\left(f(x_5)+f(x_6)\right)(x_6-x_5)$

$= \frac{1}{2}\left(f(x_{k-1})+f(x_k)\right)\Delta x_k$

$= \frac{1}{2}\left(f(x_{k-1})+f(x_k)\right)\cdot \frac{b-a}{n}$



$$\text{Area} \approx \frac{b-a}{2n}\Big[f(x_0)+f(x_1)+f(x_1)+f(x_2)+f(x_2)+f(x_3)$$
$$+\cdots+f(x_{n-1})+f(x_{n-1})+f(x_n)\Big]$$
$$= \frac{b-a}{2n}\Big[f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{n-1})+f(x_n)\Big]$$

Simpson's Rule

Fitting Parabolas between each trio of points.

$$\text{Area} \approx \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + 2y_6\right]$$

$$\Delta x = \frac{b-a}{n}$$

Error Estimates

$$|E_T| \le \frac{K(b-a)^3}{12n^2}, \quad K \ge |f''(x)| \text{ on } [a,b]$$

Find $f''(x)$. Find its max on $[a,b]$ → K

Let $f(x) = \sin(4x)$. Find a lower bound on n so that $|E_T| < .0001$ on $[0, \frac{8\pi}{3}]$

$$f'(x) = 4\cos(4x)$$

$$f''(x) = -16\sin(4x)$$

$$|f''(x)| = |-16\sin(4x)| = 16|\sin(4x)| \le 16$$

$$|E_T| \le \frac{16\left(\frac{8\pi}{3}\right)^3}{12n^2} \overset{\text{want}}{\le} .0001$$

$$16\left(\frac{8\pi}{3}\right)^3 \le (.0001)(12)n^2$$

$$\frac{16\left(\frac{8\pi}{3}\right)^3}{.0012} \le n^2$$

$$\sqrt{\frac{16\left(\frac{8\pi}{3}\right)^3}{.0012}} \le n \qquad \text{Round Up}$$

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           199.3156857
16(8π/3)^3/.0012
           7839611.684
Ans^.5
           2799.930657
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$\boxed{n = 2800 \text{ is it}}$

$$|E_M| \leq \frac{K(b-a)^3}{24n^2} \quad \text{Same } K$$

$$|E_S| \leq \frac{K(b-a)^5}{180n^4} \quad K \geq \underbrace{|f^{(4)}(x)|}_{\frac{d^4 f}{dx^4}} \text{ on } [a,b]$$

$$\int e^{x^2}\,dx$$

$$\int_0^{.01} \frac{\cos(.27x)(.3x^{1.8}-11)}{\tan(x-\frac{\pi}{28})}\,dx$$