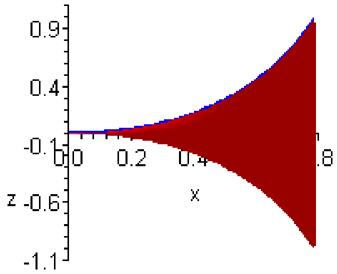
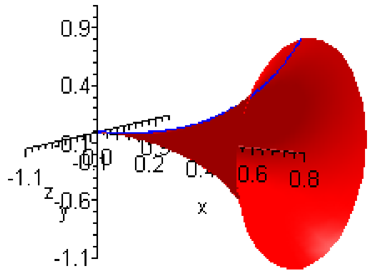
8.6 INTEGRATION USING TABLES AND COMPUTER ALGEBRA SYSTEMS





8.6 #s 5, 10, 15, 20, 25, 30, 31

$$\int_0^{\frac{1}{4}\pi} \pi \tan^4 x \, dx$$

$$= -\frac{2}{3}\pi + \frac{1}{4}\pi^2$$

$$= .373005998$$

$e^{-x} = \frac{1}{u}$

$u = e^x$

$du = e^x dx$

$$\int \frac{1}{e^x(3e^x+2)} dx = \int \frac{dx}{3e^{2x} + 2e^x} =$$

#8.6 #38:

$$\int \frac{1}{\exp(x)\cdot(3\cdot\exp(x)+2)} dx$$

$$-\frac{3}{4}x - \frac{1}{2}e^{-x} + \frac{3}{4}\ln(3e^x+2) + C$$

$$-\frac{3}{4}\ln(e^x) + \frac{3}{4}\ln(3e^x+2) - \frac{1}{2e^x}$$

simplify(%)

$$-\frac{3}{4}\ln(e^x) + \frac{3}{4}\ln(3e^x+2) - \frac{1}{2}e^{-x}$$

$$\int \frac{e^{-x}du}{3u^2+2u} = \int \frac{du}{u(3u^2+2u)}$$

$$= \int \frac{du}{u^2(3u+2)}$$

#43:

$$\int \frac{1}{x\cdot \mathrm{sqrt}(1-x^2)} dx$$

$$-\operatorname{arctanh}\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$\boxed{\int \frac{du}{u^2(3u+2)}}$$

Formula 50.

$$\left[\frac{1}{u^2(3u+2)} = \frac{A}{u} + \frac{B}{u^2} + \frac{C}{3u+2}\right] u^2(3u+2)$$

$$1 = Au(3u+2) + B(3u+2) + Cu^2$$

$x^2+5x+6 \neq$

$x^2+5x+16$

Let $u=0$

$$1 = 2B \Longrightarrow \boxed{B = \frac{1}{2}}$$

Let $u = -\frac{2}{3}$

$$1 = C\left(-\frac{2}{3}\right)^2 = \frac{4}{9}C \longrightarrow \boxed{C = \frac{9}{4}}$$

Let $u = 1$

$$5A + 5B + C = 1$$

$$5A + \frac{5}{2} + \frac{9}{4} = 1$$

$$20A + 10 + 9 = 4$$

$$20A = -15$$

$$\boxed{A = -\frac{3}{4}}$$

w/o #50, we have

$$\int\left(-\frac{3}{4}\left(\frac{1}{u}\right) + \frac{1}{2}\left(\frac{1}{u^2}\right) + \frac{9}{4}\left(\frac{1}{3u+2}\right)\right)du$$

$$= -\frac{3}{4}\ln|u| + \frac{1}{2}(-1)u^{-1} +$$

$$\frac{9}{4}\cdot\frac{1}{3}\ln|3u+2| + C$$

$v = 3u+2$

$dv = 3du$

$\int \frac{dv}{v}$

$$-\frac{3}{4}x - \frac{1}{2}e^{-x} + \frac{3}{4}\ln(3e^x+2) + C$$

$$\int \frac{du}{u^2(3u+2)}$$

FORMULA 50.

$$\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln\left|\frac{a+bu}{u}\right| + C$$

$a=2$
$b=3$

$$-\frac{1}{2u} + \frac{3}{4} \ln\left|\frac{2+3u}{u}\right| + C$$

$$= -\frac{1}{2e^x} + \frac{3}{4} \ln\left|\frac{2+3e^x}{e^x}\right| + C$$

$$= -\frac{1}{2}e^{-x} + \frac{3}{4}\ln|3e^x+2| - \frac{3}{4}\ln|e^x| + C$$

$$= -\frac{1}{2}e^{-x} + \frac{3}{4}\ln|3e^x+2| - \frac{3}{4}x + C$$

$$-\frac{3}{4}x - \frac{1}{2}e^{-x} + \frac{3}{4}\ln(3e^x+2) + C$$

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Chemical Rubber Handbook.

$$\frac{1}{2}\int 2x\sin(x^2)\cos(3x^2)\,dx = \boxed{\frac{1}{2}\int \sin(u)\cos(3u)\,du}$$

$u = x^2 \longrightarrow du = 2x\,dx$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$$

$$= \frac{1}{2}\left[\sin(-2u) + \sin(4u)\right]$$

$$= \frac{1}{2}\left[-\sin(2u) + \sin(4u)\right]$$

$$= \frac{1}{2}\int\left(-\sin(2u) + \sin(4u)\right)du \quad = \frac{1}{2}\int \frac{-\sin(2u)\cdot 2du}{2} \cdots$$

$$= \frac{1}{2}\cdot\frac{1}{2}\cos(2u) - \frac{1}{2}\cdot\frac{1}{4}\cos(4u) + C$$

FORMULA #79 $\quad a=1,\ b=3$

$$-\frac{\cos((1-3)u)}{2(1-3)} - \frac{\cos((1+3)u)}{2(1+3)} =$$

$$\frac{1}{4}\cos(-2u) - \frac{\cos(4u)}{8}$$

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$$\int \frac{\ln(x)\,dx}{x\sqrt{1+(\ln(x))^2}}$$

Let $u = 1+(\ln(x))^2$

$du = 2\ln(x)\cdot\frac{1}{x}\,dx$

$$\frac{1}{2}\int\left(1+(\ln(x))^2\right)^{-\frac{1}{2}}\cdot 2\ln(x)\cdot\frac{1}{x}\,dx$$

$$= \frac{1}{2}\int u^{-\frac{1}{2}}\,du = \frac{1}{2}\cdot\frac{2}{1}u^{\frac{1}{2}}+C = \sqrt{1+(\ln(x))^2}+C$$

Let $u=\ln(x)$. Then $du=\frac{1}{x}\,dx$. This gives

$dx = x\,du$

$$\int \frac{u\cdot x\,du}{x\sqrt{1+u^2}}$$

In §8.6 Thursday

Let $v = 1+u^2$

$dv = 2u\,du$

$$= \frac{1}{2}\int\frac{2u\,du}{\sqrt{1+u^2}}$$

$$= \frac{1}{2}\int\frac{dv}{\sqrt{v}} = \frac{1}{2}\int v^{-\frac{1}{2}}\,dv = \dots$$