8.5 STRATEGY FOR INTEGRATION

How many of these do you know by heart, already? I have a different way of remembering #13. Can you guess what it is?

TABLE OF INTEGRATION FORMULAS Constants of integration have been omitted.

1. $\cancel{\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)}$
2. $\cancel{\int \frac{1}{x}\,dx = \ln|x|}$
3. $\cancel{\int e^x\,dx = e^x}$
4. $\boxed{\int a^x\,dx = \frac{a^x}{\ln a}}$
5. $\cancel{\int \sin x\,dx = -\cos x}$
6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$
8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$
10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln|\sec x + \tan x|$
12. $\int \csc x\,dx = \ln|\csc x - \cot x|$
13. $\int \tan x\,dx = \ln|\sec x|$
14. $\int \cot x\,dx = \ln|\sin x|$
15. $\int \sinh x\,dx = \cosh x$
16. $\int \cosh x\,dx = \sinh x$
17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$

*19. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$

*20. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$

$\int \frac{du}{u}$

$\int a^x\,dx =$

$\int e^{\ln(a)\,x}\,dx$

$\boxed{= \frac{1}{\ln a}\int e^{\ln(a)x}\cdot \ln(a)\,dx}$

$= \frac{1}{\ln a}\int e^{f(x)} f'(x)\,dx$

$= \frac{1}{\ln a} e^{f(x)} + C$

$= \frac{1}{\ln a} a^x + C$

0. Take a good look. Take a sip of your drink, and reflect on it. Might be something that jumps out at you, like even/odd function on an interval symmetric about the origin. You might see something that saves you a ton of work, later (Instructors *love* scary-looking test questions that are *easy*, if you can just *see* the lever...).

1. Simplify the Integrand if Possible Sometimes the use of algebraic manipulation or trigonometric identities will simplify the integrand and make the method of integration obvious.

2. Look for an Obvious Substitution Try to find some function $u = g(x)$ in the integrand whose differential $du = g'(x)\,dx$ also occurs, apart from a constant factor.

3. Classify the Integrand According to Its Form If Steps 1 and 2 have not led to the solution, then we take a look at the form of the integrand $f(x)$.

a. Trig Functions – 8.2 skills.
b. Rational Functions – 8.4.
c. Integration by Parts – 8.1 – Look for powers of *x* times transcendental functions (, etc.)
d. Radicals – Trig substitution tricks (8.3) for situations and the "simplifying substitution" we spoke about briefly in 8.4: Use when facing situations that aren't cured by other techniques.

$$\int \arctan\sqrt{x}\,dx$$

$u=\sqrt{x}$

$du=\frac{1}{2\sqrt{x}}dx$

$dx=2\sqrt{x}\,du$

$=2u\,du$

$$=\int \arctan(u)\cdot 2u\,du$$

$$=2\int u\arctan(u)\,du$$

Let $x=u$

$$=\boxed{2\int x\arctan(x)\,dx}$$

Let $u=x \Rightarrow du=dx$

$dv=\arctan(x)dx$ so

$v=$ DOH!

Try $u=\arctan(x)$

$dv=x\,dx$

$v=\frac{1}{2}x^2$

FORMULA #92 !? Sick

$$=2\left[\frac{x^2+1}{2}\arctan(x)-\frac{x}{2}+C\right]$$

Back-track:

$$=2\frac{u^2+1}{2}\arctan(u)-2\frac{u}{2}+C$$

$$=2\frac{(\sqrt{x})^2+1}{2}\arctan\sqrt{x}-2\frac{\sqrt{x}}{2}+C$$

$2\int x \arctan(x)\,dx$

Let $u = x \Rightarrow du = dx$
$dv = \arctan(x)dx$ so
$v =$ Doh!

Try $u = \arctan(x)$ $\quad du = \frac{dx}{x^2+1}$
$dv = x\,dx$
$v = \frac{1}{2}x^2$

$= uv - \int v\,du =$

$2\left[\frac{1}{2}x^2\arctan(x) - \int \frac{1}{2}x^2 \frac{dx}{x^2+1}\right]$

$= x^2\arctan(x) - \boxed{\int \frac{x^2dx}{x^2+1}} \rightarrow I_1$

$$\begin{array}{r|l} & \;1 \quad r.\,-1 \\ x^2+1 & x^2 + 0x + 0 \\ & -(x^2 \quad\; + 1) \\ \hline & \quad -1 \end{array}$$

This says $\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$

$I_1 = \int\left(1 - \frac{1}{x^2+1}\right)dx = \int dx - \int \frac{dx}{x^2+1} = x - \arctan(x)$

This gives

$x^2\arctan(x) - [x - \underline{\arctan(x)}] + C$

$= (x^2+1)\arctan(x) - x + C$

Go back & un-substitute Blah Blah Blah

$= \boxed{(x+1)\arctan(\sqrt{x}) - \sqrt{x} + C}$

25.5

$$\int \frac{3x^2-2}{x^2-2x-8}\,dx$$

① Long division
② Partial fractions

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$$\int \sqrt{\frac{1+x}{1-x}}\ \frac{\sqrt{1-x}}{\sqrt{1-x}}\,dx$$

① Rationalize Denominator.
CRAP.

② ① Rationalize Numerator

$$\int \frac{\sqrt{1-x^2}}{1-x} \qquad \int \frac{1+x}{\sqrt{1-x^2}}\,dx$$

$$\sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = |\cos\theta| = \cos\theta$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + -\frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx \qquad u = 1-x^2 \quad du = -2x\,dx$$

$$= \arcsin(x) + -\frac{1}{2}\cdot 2\sqrt{1-x^2} + C$$

Ken says just clobber it, wimp!

$$x = \sin\theta \qquad dx = \cos\theta\,d\theta$$

$$\int \frac{1+\sin\theta}{\cos\theta}\cdot\cos\theta\,d\theta = \int (1+\sin\theta)\,d\theta$$

$$= \theta - \cos\theta + C$$

$$= \arcsin(x) - \sqrt{1-x^2} + C$$

$$u = \sqrt{\frac{1+x}{1-x}} = \frac{(1+x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}}$$

$$\ln(u) = \frac{1}{2}\ln(1+x) - \frac{1}{2}\ln(1-x)$$

$$\frac{u'}{u} = \frac{1}{2}\left(\frac{1}{1+x}\right) + \frac{1}{2}\left(\frac{1}{1-x}\right)$$

$$u' = \frac{du}{dx} = \frac{1}{2}\left[\frac{1}{1+x} + \frac{1}{1-x}\right]\sqrt{\frac{1+x}{1-x}}$$

$$du = \frac{1}{2}\left[\frac{1}{1+x} + \frac{1}{1-x}\right]\sqrt{\frac{1+x}{1-x}} \cdot dx$$

$$dx = \frac{du}{\frac{1}{2}\left[\frac{1}{1+x} + \frac{1}{1-x}\right] \cdot \sqrt{\frac{1+x}{1-x}}}$$

Train Wreck

$\int u$