§8.4 Partial Fractions

$$\frac{1}{x+2} + \frac{2}{x-3} = \frac{x-3+2x+4}{(x+2)(x-3)} = \frac{3x+1}{x^2-x-6}$$

§8.4 wants to see:

$$\int \frac{3x+1}{x^2-x-6}dx = \ldots = \int \frac{dx}{x+2} + \int \frac{2dx}{x-3} = \ln|x+2| + \ln|x-3| + C$$

$u = x+2$
$du = dx$

$u = x-3$
$du = dx$

$$\int \frac{du}{u}$$

$$\int \frac{(2x-1)dx}{x^2-x-6} = \ln|x^2-x-6| + C$$

$u = x^2-x-6$
$du = (2x-1)dx$

$$\frac{3x+1}{x^2-x-6} = \left[\frac{3x+1}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}\right](x-3)(x+2)$$

$$3x+1 = A(x+2) + B(x-3) = Ax+2A+Bx-3B$$

$3x = Ax+Bx$

$3 = A+B$

$3-A=B$

System of linear equations.

$1 = 2A-3B$

$1 = 2A-(3)(3-A)$

$1 = 2A-9+3A$

$10 = 5A$

$\boxed{2=A}$

$3-A = 3-2 = \boxed{1=B}$

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & -3 & 1 \end{array}\right] \sim$$

$$\begin{array}{c} R1 \\ -2R1+R2 \end{array} \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -5 & -5 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc} 1 & 1 & 3 \\ 0 & 1 & 1 \end{array}\right] \begin{array}{c} R1 \\ \frac{-1}{5}R2 \end{array}$$

$B = 1$

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right] \begin{array}{c} -R2+R1 \\ R2 \end{array}$$

$1A+0B = 2$

$0A+1B = 1$

$$\frac{3x^2-5x+2}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \quad \text{etc.}$$

$$\frac{3x^2-5x+2}{(x-1)(x^2+2x+5)}$$

$$= \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+5} \quad \text{etc.}$$

x^2+2x+5

$a=1, b=2, c=5$

$b^2-4ac = 4-4(1)(5)$

$= -16$

has no real zeros, i.e., is irreducible over the real number field.

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$$

is the new kid on the block.

$$x^2+2x+5$$
$$= x^2+2x+1^2-1+5$$
$$= (x+1)^2+4$$

$$\int \frac{dx}{x^2+2x+5} = \int \frac{dx}{(x+1)^2+2^2}$$

$u = x+1 \Rightarrow du = dx$

$$= \frac{1}{2}\arctan\left(\frac{x+1}{2}\right) + C$$

Improper Rational Functions.

$$\int \frac{x^3}{(x-2)(x+3)}dx = \int \frac{x^3}{x^2+x-6}dx = \int (x-1)dx + \boxed{\int \frac{7x-6}{x^2+x-6}dx} \text{ etc.}$$

will take partial fractions.

$$\begin{array}{r|l}
 & x \quad -1 \quad r\ 7x-6 \\
\hline
x^2+x-6 & x^3+0x^2+0x+0 \\
 & -(x^3+x^2-6x) \\
\hline
 & -x^2+6x+0 \\
 & -(-x^2-x+6) \\
\hline
 & 7x-6
\end{array}$$

(20) $\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx$

$\int \frac{x^2-5x+6}{(2x+1)(x-2)^2}\,dx = \int \frac{x-3}{(2x+1)(x-2)}\,dx$

Typo

$$\left(\frac{x^2-5x+6}{(2x+1)(x-2)^2} = \frac{A}{2x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}\right)(2x+1)(x-2)^2$$

$$x^2-5x+6 = A(x-2)^2 + B(x-2)(2x+1) + C(2x+1)$$

$x=1$: $A + B(-1)(3) + 3C = 2$

$A - 3B + 3C = 2$

$x=0$: $4A - 2B + C = 6$

Another approach.

Standard idiot approach

$x^2-5x+6 = A(x-2)^2 + B(x-2)(2x+1) + C(2x+1)$

$x^2-5x+6 = A(x^2-4x+4) + B(2x^2-3x-2) + 2Cx + C$

$= Ax^2 - 4Ax + 4A + 2Bx^2 - 3Bx - 2B + 2Cx + C$

$1 = A + 2B \Rightarrow A = 1-2B$

$-5 = -4A - 3B + 2C \Rightarrow -5 = -4(1-2B) - 3B + 2C$

$6 = 4A - 2B + C \Rightarrow 6 = 4(1-2B) + C - 2B$

$-4 + 8B - 3B + 2C = -5 \Rightarrow$ $\boxed{\begin{array}{l} 5B + 2C = -1 \\ -10B + C = 2 \end{array}}$

$-2B + 4 - 8B + C = 6 \Rightarrow$

$-10(-\frac{1}{5}) + C = 2$

$2 + C = 2$

$C = 0$!?

$C = 2 + 10B \Rightarrow$

$5B + 2(2+10B) = -1$

$5B + 4 + 20B = -1$

$25B = -5$

$\boxed{B = -\frac{1}{5}}$

$x^2-5x+6 = A(x-2)^2 + B(x-2)(2x+1) + C(2x+1)$

Let $x=2$ Sometimes makes it quick

$2^2-5(2)+6$

$0 = C(2(2)+1) = 5C \Rightarrow C=0.$ Nope

Newp

Sometimes it screws you up.

Not the fault of the method. Fault is in the person.

8.4
#20

$$\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx = \int\left(\frac{A}{2x+1}+\frac{B}{x-2}+\frac{C}{(x-2)^2}\right)dx$$

$$x^2-5x+16 = A(x-2)^2 + B(2x+1)(x-2) + C(2x+1)$$
$$= A(x^2-4x+4) + B(2x^2-3x-2) + 2Cx + C$$
$$= Ax^2 - 4Ax + 4A + 2Bx^2 - 3Bx - 2B + 2Cx + C$$

x^2

$\Rightarrow A+2B=1 \Rightarrow A=1-2B \rightarrow A=3$

x

$-4A-3B+2C=-5 \Rightarrow -4(1-2B)-3B+2C=-5$

$-4+8B-3B+2C=-5$

$\boxed{5B+2C=-1}$

$4A-2B+C=16 \Rightarrow 4(1-2B)-2B+C=16$

$C=2$

$4-8B-2B+C=16$

$\boxed{-10B+C=12}$

$C=10B+12$

$5B+2(10B+12)=5B+20B+24=-1$

$25B=-25$

$\boxed{B=-1}$

8.3 #16

$$\int_{\frac{\sqrt{2}}{3}}^{\frac{2}{3}} \frac{dx}{x^5\sqrt{9x^2-1}} = \int_{\frac{\sqrt{2}}{3}}^{\frac{2}{3}} \frac{dx}{x^5\cdot 3\sqrt{x^2-\frac{1}{9}}} = \frac{1}{3}\int_{\frac{\sqrt{2}}{3}}^{\frac{2}{3}} \frac{dx}{x^5\sqrt{x^2-\frac{1}{9}}}$$

$a=\frac{1}{3}$ $\quad x=\frac{1}{3}\sec\theta$

$dx=\frac{1}{3}\sec\theta\tan\theta\, d\theta$

$x=\frac{1}{3}\sec\theta=\frac{\sqrt{2}}{3}$

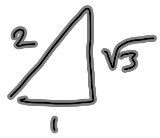
$\sec\theta=\sqrt{2}$

$\theta=\frac{\pi}{4}$

$x=\frac{1}{3}\sec\theta=\frac{2}{3}$

$\sec\theta=2$

$\theta=\frac{\pi}{3}$



$$3^5\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\frac{1}{3}\sec\theta\tan\theta\, d\theta}{\sec^5\theta\,\tan\theta}$$

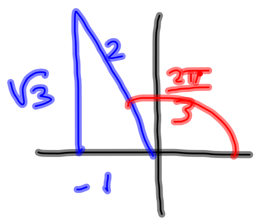
$$= 3^4\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{d\theta}{\sec^4\theta} = 3^4\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos^4\theta\, d\theta$$

$$(\cos^2\theta)^2 = \left(\frac{1+\cos 2\theta}{2}\right)^2 = \frac{1}{4}\left(1+2\cos(2\theta)+\cos^2(2\theta)\right)$$

$$= \frac{1}{4}+\frac{1}{2}\cos(2\theta)+\frac{1}{4}\left(\frac{1}{2}(1+\cos(4\theta)\right)$$

$$= \frac{1}{4}+\frac{1}{2}\cos(2\theta)+\frac{1}{8}+\frac{1}{8}\cos(4\theta) \quad \text{This gives}$$

$$3^4\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\left(\frac{3}{8}+\frac{1}{2}\cos(2\theta)+\frac{1}{8}\cos(4\theta)\right)d\theta$$

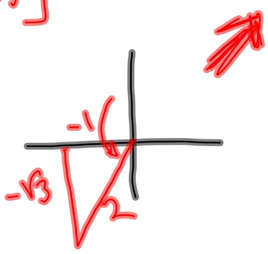


$$= 3^4\left[\frac{3}{8}\theta+\frac{1}{4}\sin(2\theta)+\frac{1}{32}\sin(4\theta)\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= 3^4\left[\frac{3}{8}\left(\frac{\pi}{3}-\frac{\pi}{4}\right)+\frac{1}{4}\left(\sin\left(\frac{2\pi}{3}\right)-\sin\left(\frac{\pi}{2}\right)\right)+\frac{1}{32}\left(\sin\left(\frac{4\pi}{3}\right)-\sin(\pi)\right)\right]$$

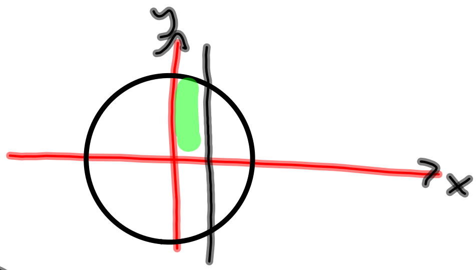
$$= 3^4\left[\frac{3}{8}\cdot\frac{\pi}{12}+\frac{1}{4}\left(\frac{\sqrt{3}}{2}-1\right)+\frac{1}{32}\left(-\frac{\sqrt{3}}{2}-0\right)\right]$$

$=$



§8.3 #42

$$2\int_0^2 \sqrt{25-x^2}\,dx$$



$x=2$

Use #35 trick for this.

result.

$$\int \frac{4x-4}{x^2+4}\,dx$$

$$= \int \frac{4x\,dx}{x^2+4} - \int \frac{4\,dx}{x^2+4}$$

$$= 2\int \frac{2x\,dx}{x^2+4} - 4\int \frac{dx}{x^2+2^2}$$

$$= 2\int \frac{du}{u} - 4 \cdot \frac{1}{2}\arctan\left(\frac{x}{2}\right)$$

$$= 2\ln|x^2+4| - 2\arctan\left(\frac{x}{2}\right) + C$$

Sep 23-12:49 PM