§8.4 Partial Fractions

$$\frac{1}{x+2} + \frac{2}{x-3} = \frac{x-3+2x+4}{(x+2)(x-3)} = \frac{3x+1}{x^2-x-6}$$

§8.4 wants to see:

$$\int \frac{3x+1}{x^2-x-6}dx = \ldots = \int \frac{dx}{x+2} + \int \frac{2dx}{x-3} = \ln|x+2| + \ln|x-3| + C$$

$u=x+2$
$du=dx$

$u=x-3$
$du=dx$

$$\int \frac{du}{u}$$

$$\int \frac{(2x-1)dx}{x^2-x-6} = \ln|x^2-x-6|+C$$

$u=x^2-x-6$
$du=(2x-1)dx$

$$\frac{3x+1}{x^2-x-6} = \left[\frac{3x+1}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}\right](x-3)(x+2)$$

$$3x+1 = A(x+2) + B(x-3) = Ax+2A+Bx-3B$$

$3x = Ax+Bx$

$3 = A+B$

$3-A=B$

$1 = 2A-3B$

$1=2A-(3)(3-A)$

$1=2A-9+3A$

$10=5A$

$\boxed{2=A}$

$3-A=3-2=\boxed{1=B}$

System of linear equations.

$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & -3 & 1 \end{array}\right] \sim$

$\begin{array}{c} R1 \\ -2R1+R2 \end{array}\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -5 & -5 \end{array}\right] \sim$

$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & 1 \end{array}\right]\begin{array}{c} R1 \\ \frac{-1}{5}R2 \end{array}$

$B=1$

$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]\begin{array}{c} -R2+R1 \\ R2 \end{array}$

$1A+0B=2$

$0A+1B=1$

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$$\frac{3x^2-5x+2}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \quad \text{etc.}$$

$$\frac{3x^2-5x+2}{(x-1)(x^2+2x+5)}$$

$$= \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+5} \quad \text{etc.}$$

x^2+2x+5

$a=1, b=2, c=5$

$$b^2-4ac = 4-4(1)(5)$$
$$=-16$$

has no real zeros, i.e., is irreducible over the real number field.

$$\int \frac{dx}{x^2+2^2} = \frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$$

is the new kid on the block.

x^2+2x+5

$= x^2+2x+1^2-1+5$

$= (x+1)^2+4$

$$\int \frac{dx}{x^2+2x+5} = \int \frac{dx}{(x+1)^2+2^2}$$

$u = x+1 \Rightarrow du = dx$

$$= \frac{1}{2}\arctan\left(\frac{x+1}{2}\right)+C$$

Improper Rational Functions.

$$\int \frac{x^3}{(x-2)(x+3)}dx = \int \frac{x^3}{x^2+x-6}dx = \int (x-1)dx + \boxed{\int \frac{7x-6}{x^2+x-6}dx} \text{ etc.}$$

will take partial fractions.

$$\begin{array}{r}
x - 1 \quad r\ 7x-6 \\
x^2+x-6 \overline{\big) \; x^3+0x^2+0x+0} \\
-(x^3+x^2-6x) \\
\hline
-x^2+6x+0 \\
-(-x^2-x+6) \\
\hline
7x-6
\end{array}$$

20 $\displaystyle\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx$

$$\left(\frac{x^2-5x+6}{(2x+1)(x-2)^2} = \frac{A}{2x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}\right)(2x+1)(x-2)^2$$

$$x^2-5x+6 = A(x-2)^2 + B(x-2)(2x+1) + C(2x+1)$$

$x=1$: $A + B(-1)(3) + 3C = 2$

$A - 3B + 3C = 2$

$x=0$: $4A - 2B + C = 6$

Another approach.

Standard idiot approach

$x^2-5x+6 = A(x-2)^2 + B(x-2)(2x+1) + C(2x+1)$

$x^2-5x+6 = A(x^2-4x+4) + B(2x^2-3x-2) + 2Cx + C$

$= Ax^2 - 4Ax + 4A + 2Bx^2 - 3Bx - 2B + 2Cx + C$

$1 = A + 2B \Longrightarrow A = 1-2B$

$-5 = -4A - 3B + 2C \Longrightarrow -5 = -4(1-2B) - 3B + 2C$

$6 = 4A - 2B + C \Longrightarrow 6 = 4(1-2B) + C - 2B$

$-4 + 8B - 3B + 2C = -5 \Longrightarrow$

$-2B + 4 - 8B + C = 6 \Longrightarrow$

$$\boxed{\begin{aligned} 5B + 2C &= -1 \\ -10B + C &= 2 \end{aligned}}$$

$-10(-\frac{1}{5}) + C = 2$

$2 + C = 2$

$C = 0$!?

$C = 2 + 10B \Longrightarrow$

$5B + 2(2+10B) = -1$

$5B + 4 + 20B = -1$

$25B = -5$

$\boxed{B = -\frac{1}{5}}$

$x^2-5x+6 = A(x-2)^2 + B(x-2)(2x+1) + C(2x+1)$

$2^2-5(2)+6$ Let $x=2$ Sometimes makes it quick

$0 = C(2(2)+1) = 5C \Rightarrow C=0.$ Nope

Newp

Sometimes it screws you up.

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