(30) $\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr$

$u = r^2 \Longrightarrow du = 2r\,dr$

$dv = \frac{r}{\sqrt{4+r^2}}\,dr$

$\Longrightarrow v = \sqrt{4+r^2}$

$\int u\,dv = uv - \int v\,du$

So, $\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr$

$= (r^2)(4+r^2)^{\frac{1}{2}}\Big]_0^1 - \int_0^1 (\sqrt{4+r^2})(2r\,dr)$

$= (1^2)(\sqrt{4+1^2}) - [0^2(\dots)]$

$- \frac{2}{3}\left[5^{3/2} - 4^{3/2}\right]$

$= \sqrt{5} - \frac{2}{3}\left[5\sqrt{5} - 8\right]$

$5^{\frac{3}{2}} = 5^{1+\frac{1}{2}} = 5^1 5^{\frac{1}{2}} = 5\sqrt{5}$

$= \sqrt{5} - \frac{10}{3}\sqrt{5} + \frac{16}{3}$

$= -\frac{7}{3}\sqrt{5} + \frac{16}{3}$

$\frac{1}{2}\int \frac{2r}{\sqrt{4+r^2}}\,dr$

$= \frac{1}{2}\int u^{-\frac{1}{2}}\,du$

$= \frac{1}{2}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= u^{\frac{1}{2}} + C$

Let $t = 4+r^2$

$\Longrightarrow dt = 2r\,dr$

$r=0 \Longrightarrow t=4$

$r=1 \Longrightarrow t=5$

$\int_4^5 t^{\frac{1}{2}}\,dt$

$= \frac{2}{3}t^{\frac{3}{2}}\Big]_4^5$

$= \frac{2}{3}\left[5^{\frac{3}{2}} - 4^{\frac{3}{2}}\right]$

$$\frac{d}{dx}\left[3^{10x}\right] = \frac{d}{dx}\left[\left(e^{\ln 3}\right)^{10x}\right]$$

$$= \frac{d}{dx}\left[e^{(\ln 3)(10)x}\right] = e^{(\ln 3)(10)x} \cdot \frac{d}{dx}\left[(\ln 3)(10)x\right]$$

$$= \left(e^{\ln 3}\right)^{10x} \cdot (\ln 3)(10)$$

$$(10 \ln 3)(3^{10x})$$

$$\frac{d}{dx}\left[3^{x}\right] = (\ln 3)(3^{x})$$

$$\frac{d}{dx}\left[3^{f(x)}\right] = \ln(3) \cdot 3^{f(x)} \cdot f'(x)$$

$$\cancel{\int e^{x}\,dx = \frac{1}{x+1} e^{x+1} + C}$$

8.2 TRIGONOMETRIC INTEGRALS

Cheat Sheet Stuff:

Pythagorean Trig Identities for sine, cosine, tangent, and their evil twins

Sum, Difference and Half-Angle formulas for sine and cosine.

8.2 #s 6, 9, 12, 18, 27, 28, 37, 38, 44, 55, 62

$$\sin^2 x = \frac{1}{2}\left[1 - \cos(2x)\right]$$

$$\cos^2 x = \frac{1}{2}\left[1 + \cos(2x)\right]$$

$$\left.\begin{aligned} \sin A \cos B &= \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right] \\ \sin A \sin B &= \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right] \\ \cos A \cos B &= \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right] \end{aligned}\right\} \text{Pg 501}$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sin x \cos x \, dx = \frac{1}{2}\sin^2 x + C$$

$u = \sin x$

$du = \cos x \, dx$

$$\int u \, du = \frac{1}{2}u^2 + C$$

$u = \cos x$

$du = -\sin x \, dx$, so

$$-\int \cos x \, (-\sin x \, dx)$$

$$= -\frac{1}{2}\cos^2 x + C$$

!?

$$= -\frac{1}{2}(1 - \sin^2 x) + C$$

$$= -\frac{1}{2} + \frac{1}{2}\sin^2 x + C$$

$$\int \sin^5 x \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$\int u^5 du = \frac{1}{6}\sin^6 x + C$$

$$\int \tan^{13} x \sec^2 x \, dx$$

$u = \tan x$
$du = \sec^2 x \, dx$

$$= \int u^{13} \, du$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int \frac{\sec^2 x \, dx}{\tanh(x) - 7}$$

$u = \tanh(x)$
$du = \operatorname{sech}^2(x) \, dx$

$$\int \frac{du}{u-7}$$

$$\int \frac{du}{u-7}$$

Let $v = u - 7$
$dv = du$

$$= \int \frac{dv}{v} = \ln|v| + C$$
$$= \ln|u-7| + C$$
$$= \ln|\tanh(x) - 7| + C$$

L
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$$\int \sin^5 x \cos^3 x \, dx = \int \sin^5(x)\left(1-\sin^2(x)\right)\cos(x)\,dx$$

$$= \int \sin^5(x)\cos(x)\,dx - \int \sin^7(x)\cos(x)\,dx$$

$u = \sin(x)$
$du = \cos(x)\,dx$

EVIL PERSON:

$$= \int \left(1-\cos^2(x)\right)^2 \cdot \cos^3(x) \cdot \sin(x)\,dx$$

$$= \int \left(1-2\cos^2(x) + \cos^4(x)\right)\cos^3(x)\cdot\sin(x)\,dx$$

etc.

$\sin^4(x) = \left(\sin^2(x)\right)^2$
$= \left(1-\cos^2(x)\right)^2$

$u = \cos(x)$
$du = -\sin(x)\,dx$

$\int \sin^5 x \, dx$ is hard, w/o a $du = \cos x \, dx$ to help.

5 causes heartburn

$\boxed{\int \sin^4 x \, dx}$ see Example 4.

$$\int \underbrace{\sin^4 x} \cdot \sin x \, dx$$

$$= \int (\sin^2 x)^2 \sin x \, dx$$

$$= \int (1-\cos^2 x)^2 \sin x \, dx$$

$$= \int (1 - 2\cos x + \cos^2 x)\sin x \, dx$$

$$= \int \sin x \, dx - 2\int \cos x \sin x \, dx + \int \cos^2 x \sin x \, dx$$

$$= -\cos(x) + 2\int \cos(x)\left(-\sin(x)\,dx\right) - \int \cos^2(x)\left(-\sin(x)\,dx\right)$$

$$2\int u\,du \qquad - \int u^2\,du$$

etc.

$$\boxed{\int \sin^2 x \, dx = \frac{1}{2}\int (1-\cos(2x))\,dx}$$

$$= \frac{1}{2}\int dx - \frac{1}{2}\cdot\frac{1}{2}\int \cos(2x)\cdot 2\,dx$$

$u = 2x$

$du = 2dx$

$dx = \frac{1}{2}du$

$$= \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$$

$$\int \sin^4(x)\,dx = \int \left(\sin^2(x)\right)^2 dx$$

$$= \int \left(\frac{1}{2}(1-\cos(2x)\right)^2 dx$$

$$= \frac{1}{4}\int \left(1 - 2\cos(2x) + \cos^2(2x)\right) dx$$

$$= \frac{1}{4}\int dx - \frac{1}{2}\int \cos(2x)\,dx + \frac{1}{4}\int \cos^2(2x)\,dx$$

$$= \frac{1}{4}x - \frac{1}{2}\cdot\frac{1}{2}\int \cos(2x)\cdot 2dx + \frac{1}{4}\int \frac{1}{2}(1+\cos(4x))\,dx$$

etc.

$$\int x \sin^2(x)\,dx$$

$u = x \implies du = dx$

$\int dv = \int \sin^2(x)\,dx$

$= \frac{1}{2}\int (1-\cos(2x))\,dx$

$\implies v = \frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{2}\sin(2x)$

$= uv - \int v\,du$

$= x\cdot\left(\frac{1}{2}x - \frac{1}{4}\sin(2x)\right)$

$- \int\left(\frac{1}{2}x - \frac{1}{4}\sin(2x)\right)dx = \text{etc.}$

$u=2x$

$-\int \cos(2x)\,dx$ $\quad u = 2x$, $du = 2dx$, $dx = \frac{du}{2}$

$= -\frac{1}{2}\int \cos(2x)\cdot 2dx$

$= -\frac{1}{2}\int \cos(u)\,du$

$$\int_0^{\frac{\pi}{6}} x \sin(x)\,dx \qquad \begin{array}{l} u = x \to du = dx \\ dv = \sin(x)dx \to v = -\cos(x) \end{array}$$

$$= uv - \int v\,du = -x\cos(x)\Big|_0^{\frac{\pi}{6}} + \int_0^{\frac{\pi}{6}} \cos(x)\,dx$$

$$= -\frac{\pi}{6}\cos\left(\frac{\pi}{6}\right) + \sin(x)\Big|_0^{\frac{\pi}{6}} = -\frac{\pi}{6}\cdot\frac{\sqrt{3}}{2} + \left[\frac{1}{2} - 0\right]$$

$$= \frac{-\sqrt{3}\,\pi}{12} + \frac{1}{2} = \frac{6 - \sqrt{3}\,\pi}{12}$$

$$= uv - \int v\,du = -x\cos(x)\Big|_0^{\frac{\pi}{6}} + I_2$$

$$I_2 : \int_0^{\frac{\pi}{6}} \cos(x)\,dx$$

Another way

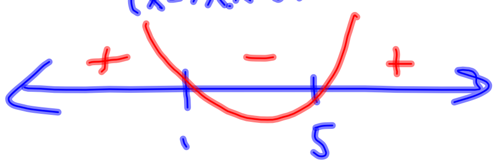
$$= \Big[-x\cos(x) + \sin(x)\Big]_0^{\frac{\pi}{6}}$$

Sep 19-12:50 PM

$\ln(x^2-6x+5)$

Need $x^2-6x+5>0$

$(x-1)(x-5)>0$



$(-\infty,1)\cup(5,\infty)$

$$\boxed{\begin{array}{c}(x-1)(x-5)>0\\ \cancel{x-1>0 \text{ OR } x-5>0}\end{array}}$$

→ Bull

$x^2-6x+5>0$

$x^2-6x \quad >-5$

$x^2-6x+3^2>-5+9$

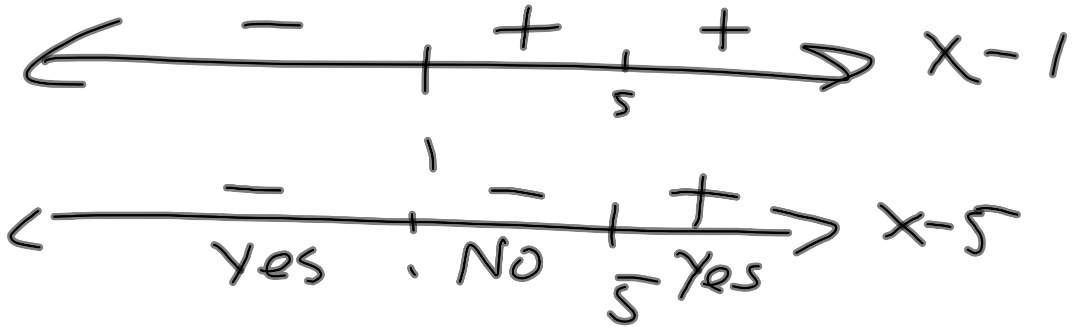
$(x-3)^2>4$

$\sqrt{(x-3)^2}>\sqrt{4}$

$|x-3|>2$

$x-3>2$ OR $x-3<-2$

$x>5$ OR $x<1$



$\int x \cdot \cos(x)^2 \, dx$

$$x \left(\frac{1}{2} \cos(x) \sin(x) + \frac{1}{2} x \right) + \frac{1}{4} \cos(x)^2 - \frac{1}{4} x^2$$

$expand(\%)$

$$\frac{1}{2} x \cos(x) \sin(x) + \frac{1}{4} x^2 + \frac{1}{4} \cos(x)^2$$

$\% - \left(\frac{1}{4} x^2 + \frac{1}{4} x \cdot \sin(2x) + \frac{1}{8} \cos(2x) \right)$

$$\frac{1}{2} x \cos(x) \sin(x) + \frac{1}{4} \cos(x)^2 - \frac{1}{4} x \sin(2x) - \frac{1}{8} \cos(2x)$$

$expand(\%)$

$$\frac{1}{8}$$

$simplify(\%)$

$$x \cos(x) \sin(x) + \frac{1}{2} \cos(x)^2 - \frac{1}{8}$$