$$(fg)' = f'g + fg'$$

$$f'g + fg' = (fg)'$$

$$fg' = (fg)' - f'g$$

$$\int fg' = \int (fg)' - \int f'g$$

$$\int fg' = fg - \int f'g$$

~~Integreat Grateful Dead~~

Let $u = f(x) \Longrightarrow du = f'(x)\,dx$

$dv = g'(x)\,dx \Longrightarrow v = g(x)$

$$\int u\,dv = uv - \int v\,du$$

Integration by Parts Formula.

Something that vanishes by differentiation

Something you can integrate times dx

$$\int x \sin(x)\,dx \qquad \left(\begin{array}{l} \text{Let } u = x \Rightarrow du = dx \\ dv = \sin(x)\,dx \Rightarrow v = -\cos(x) \end{array} \right)$$

$$= uv - \int v\,du$$

$$= x\left(-\cos(x)\right) - \int \left(-\cos(x)\right) dx$$

$$= -x\cos(x) + \int \cos(x)\,dx = -x\cos(x) + \sin(x) + C$$

$\int x^2 \sin(x)\,dx$ takes 2 integrations by parts

$\int x^5 \sin(x)\,dx$ " 5 " " "

Classic Application See Example 4

My u & dv will be different.

$\int e^x \sin(x)\,dx$ (Let $u=e^x \Rightarrow du=e^x dx$
$dv=\sin(x)\,dx \Rightarrow v=-\cos(x)$)

$= uv-\int v\,du$

$= e^x(-\cos(x)) - \int(-\cos(x))(e^x dx)$

$u=e^x \rightarrow du=e^x dx$
$dv=\cos(x)dx$
$\Rightarrow v=\sin(x)$

$=-e^x\cos(x) + \int e^x\cos(x)\,dx$

$=-e^x\cos(x) + uv-\int v\,du$

$=-e^x\cos(x)+e^x\sin(x)-\int e^x\sin(x)\,dx$

$$\int e^x\sin(x)\,dx = -e^x(\cos(x)-\sin(x)) - \underline{\int e^x\sin(x)\,dx}$$

$$2\int e^x\sin(x)\,dx = -e^x(\cos(x)-\sin(x))$$

$$\int e^x\sin(x)\,dx = \frac{e^x(\sin(x)-\cos(x))}{2} + C$$

Makes it more general, since C = Anything still works.

Write Much, Think Little

$= \frac{1}{2}\left[e^x\sin(x)-e^x\cos(x)\right]$. Differentiate.

$$\frac{d}{dx}\left[\text{the above}\right] = \frac{1}{2}\Big[\overset{f'}{e^x}\overset{g}{\sin(x)} \overset{+f}{+e^x}\overset{g'}{(\cancel{+}\cos(x))} - e^x\cos(x) - e^x(-\sin x)\Big]$$

$= \frac{1}{2}\left[2e^x\sin(x)\right] = \checkmark$

Sep 16-12:21 PM

$\frac{d}{dx}\left[\ln(x)\right] = \frac{1}{x}$ makes $\ln(x)$ a good candidate for u.

$$\int_1^2 x^3 \ln(x)\,dx$$

Let $u = \ln(x) \Longrightarrow du = \frac{dx}{x}$

$dv = x^3 dx \Longrightarrow v = \frac{1}{4}x^4$

$$= uv - \int v\,du = \frac{1}{4}x^4 \ln(x)\Big|_1^2 - \frac{1}{4}\int_1^2 x^4 \cdot \frac{1}{x}\,dx$$

$$= \frac{1}{4}\left[2^4 \ln(2) - 1^4 \ln(1)\right] - \frac{1}{4}\int_1^2 x^3\,dx$$

$$= \frac{1}{4}\cdot 16 \ln(2) - \frac{1}{4}\cdot\frac{1}{4}x^4\Big|_1^2 = 4\ln(2) - \frac{1}{16}\left[16 - 1\right]$$

$$= \boxed{4\ln(2) - \frac{15}{16}}$$

psi

Φ

ϕ

$\alpha, \beta, \gamma\ \delta$

A, B, Γ, Δ

$$\int \frac{r^3}{\sqrt{r^2+4}}\,dr$$

$u = r^3$

$dv = \frac{1}{\sqrt{r^2+4}}\,dr$

$= (r^2+4)^{-\frac{1}{2}}\,dr$

But to antidifferentiate, by power rule, I'd need

$(r^2+4)^{-\frac{1}{2}}(\underline{2r\,dr})$

$= u^{-\frac{1}{2}}\,du$

Borrow an r from r^3

$u = r^2 \Longrightarrow du = 2r\,dr$

$\frac{1}{2} \cdot 2\,(r^2+4)^{-\frac{1}{2}} \cdot r\,dr$

§8.1 #s 2, 4, 8, 10, 17, 24, 30, 33, 34

Wheedling sucks. ~~Wheedle~~