7.3 #s 2, 4, 7, 10, 18, 22, 23, 26, 28, 22, 36, 50, 54, 60, 62, 65

See website for Chapter 7 homework assignments.

3–8 Find the exact value of each expression.

3. (a) $\log_5 125 =$

$\log_5(5^3) = 3$

$3\log_5(5^1) = 3 \cdot 1$

(b) $\log_3 \frac{1}{27}$

$= \log_3\left(\frac{1}{3^3}\right)$

$= \log_3(3^{-3}) = -3$

7. (a) $\log_2 6 - \log_2 15 + \log_2 20 = \log_2\left(\frac{\cancel{6}^{2} \cdot \cancel{20}^{4}}{\cancel{15}\,\cancel{5}\,1}\right) = \log_2(8) = 3$

(b) $\log_3 100 - \log_3 18 - \log_3 50$

$= \log_3\left(\frac{100}{18 \cdot 50}\right) = \log_3\left(\frac{2}{18}\right) = \log_3\left(\frac{1}{9}\right) = -2$

$(ab)^c = a^c b^c$

$(a^b a^c) = a^{b+c}$

$(a^b)^c = a^{bc}$

$\frac{a^b}{a^c} = a^{b-c}$

$\log(ab) = \log a + \log b$

$\log(a^b) = b \log a$

$\log\left(\frac{a}{b}\right) = \log a - \log b$

etc.

9–12 Use the properties of logarithms to expand the quantity.

9. $\log_2\left(\frac{x^3 y}{z^2}\right)$

(b) What is the domain of this function?

(c) What is the range of this function?

(d) Sketch the general shape of the graph of the function $y = \log_a x$ if $a > 1$.

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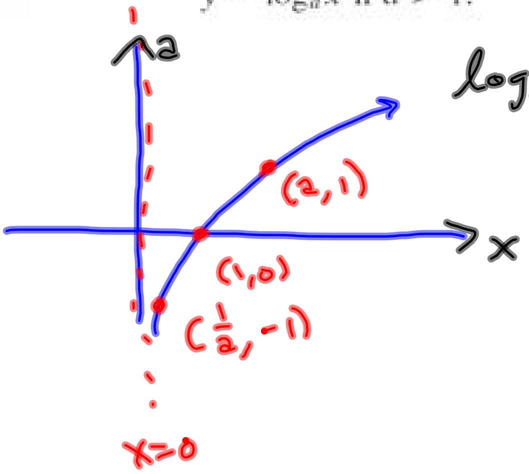


$\log_a(x)$ for $a > 1$

$\log_a(a^x) = x$

$a^{\log_a(x)} =$

13–18 Express the quantity as a single logarithm.

13. $\log_{10} a - \log_{10} b + \log_{10} c$

$= \log\left(\frac{ac}{b}\right)$

$3\log(x) + 17\log(y) - 5\log(z)$

$=$

$= \log\left(\frac{x^3y^{17}}{z^5}\right)$

General

4 If $a > 1$, then

$$\lim_{x \to \infty} \log_a x = \infty \qquad \text{and} \qquad \lim_{x \to 0^+} \log_a x = -\infty$$

$$\log_e x = \ln x$$

5 $$\ln x = y \iff e^y = x$$

Meh

6 $$\ln(e^x) = x \qquad x \in \mathbb{R}$$
$$e^{\ln x} = x \qquad x > 0$$

Yeah. Always think in terms of their being inverses.

$$\ln e = 1$$

7 **CHANGE OF BASE FORMULA** For any positive number a ($a \neq 1$), we have

$$\log_a x = \frac{\ln x}{\ln a}$$

$$\log_3(17) = \frac{\ln(17)}{\ln(3)}$$

Calculator.

19. Use Formula 7 to evaluate each logarithm correct to six decimal places.
(a) $\log_{12} e$ (b) $\log_6 13.54$ (c) $\log_2 \pi$

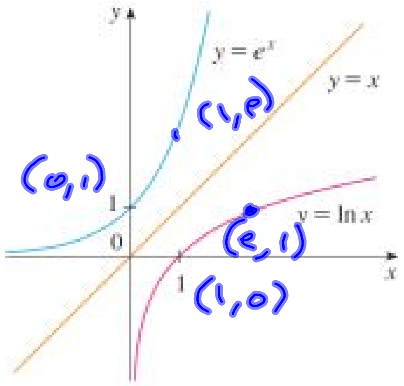


FIGURE 3

20–22 Use Formula 7 to graph the given functions on a common screen. How are these graphs related?

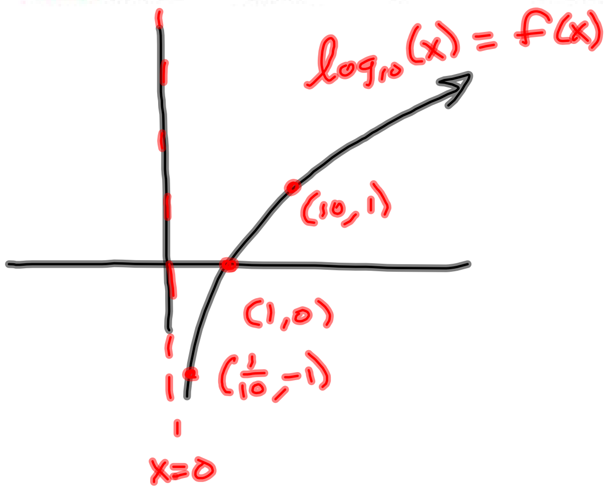
20. $y = \log_2 x$, $y = \log_4 x$, $y = \log_6 x$, $y = \log_8 x$

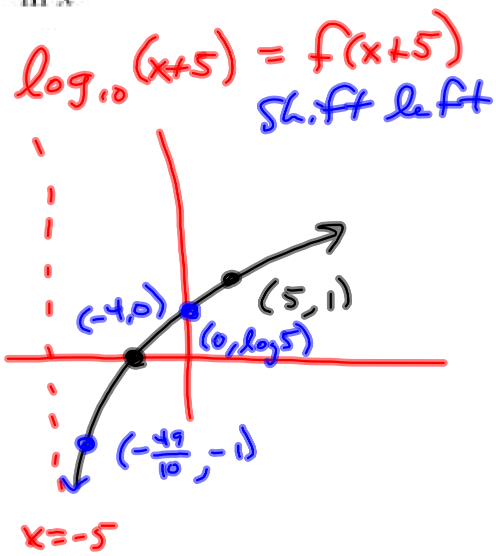
$$y = \frac{\ln(x)}{\ln(2)}$$

$Y_1 = \ln(x)/\ln(2)$
etc.

23–24 Make a rough sketch of the graph of each function. Do not use a calculator. Just use the graphs given in Figures 2 and 3 and, if necessary, the transformations of Section 1.3.

$\boxed{23.}$ (a) $y = \log_{10}(x + 5)$ (b) $y = -\ln x$





25–34 Solve each equation for x.

26. (a) $e^{2x+3} - 7 = 0$

$\S$7.3 Due Tues/Wed.

$\S$7.2 .. Monday

$$e^{2x+3} = 7$$

$$\ln\left(e^{2x+3}\right) = \ln(7)$$

$$2x+3 = \ln(7)$$

$$2x = \ln(7) - 3$$

$$\boxed{x = \frac{\ln(7)-3}{2}}$$

29. $3xe^x + x^2e^x = 0$

$e^x(3x+x^2) = e^x(x^2+3x) = 0$

$e^x = 0$ OR $x^2+3x=0$

Nevah!

$x(x+3)=0$

$\boxed{x=0 \text{ OR } x=-3}$

7.3 #65

$f(x) = \ln\left(x + \sqrt{x^2+1}\right)$

$\ln(2e) = \ln 2 + \ln e$
$= \ln 2 + 1$

$f(-x) = \ln\left(-x + \sqrt{(-x)^2+1}\right)$

$= \ln\left(\sqrt{x^2+1} - x\right) = f(-x)$

This is odd

$-f(x) = -\ln\left(x + \sqrt{x^2+1}\right) = \ln\left(\frac{1}{x+\sqrt{x^2+1}}\right)$

$= \ln\left(\left(x+\sqrt{x^2+1}\right)^{-1}\right)$

$= \ln\left(\frac{x-\sqrt{x^2+1}}{\left(x+\sqrt{x^2+1}\right)\left(x-\sqrt{x^2+1}\right)}\right) = \ln\left(\frac{x-\sqrt{x^2+1}}{x^2-\left(\sqrt{x^2+1}\right)^2}\right)$

$= \ln\left(\frac{x-\sqrt{x^2+1}}{x^2-x^2-1}\right) = \ln\left(\frac{x-\sqrt{x^2+1}}{-1}\right) = \ln\left(\sqrt{x^2+1} - x\right)$

$= f(-x)$

35–36 Find the solution of the equation correct to four decimal places.

35. (a) $e^{2+5x} = 100$

$\ln(e^{2+5x}) = \ln(100)$

$5x+2 = \ln(100)$

$5x = \ln(100) - 2$

$x = \dfrac{\ln(100)-2}{5}$

$\boxed{\approx .5210}$

```
(ln(100)-2)/5
        .5210340372
```

(b) $\ln(e^x - 2) = 3$

$e^{\ln(e^x-2)} = e^3$

$e^x - 2 = e^3$

$e^x = e^3 + 2$

$\ln(e^x) = \ln(e^3+2)$

$x = \ln(e^3+2)$

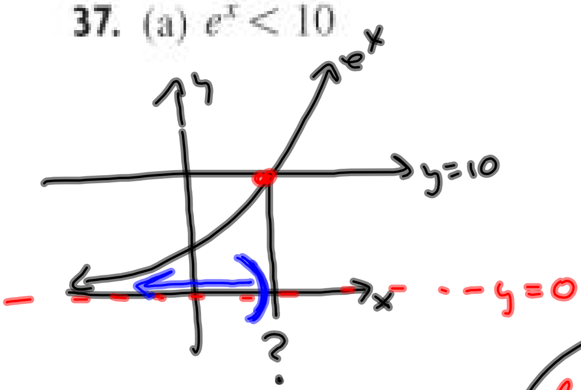
≈ 3.0949

```
(ln(100)-2)/5
        .5210340372
ln(e^(3)+2)
        3.094922956
```

You can always just solve it like an equation, and then test the sign/size of the expressions on the intervals between the solutions.

37–38 Solve each inequality for x.

37. (a) $e^x < 10$



e^x is increasing.

$e^x < 10$

$\ln(e^x) < \ln(10)$

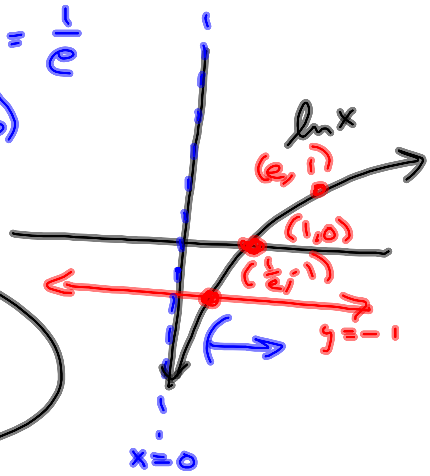
$x < \ln(10)$

$(-\infty, \ln(10))$

(b) $\ln x > -1$

$x > e^{-1} = \frac{1}{e}$

$(\frac{1}{e}, \infty)$



$\ln(x)$ is increasing
e^x " "

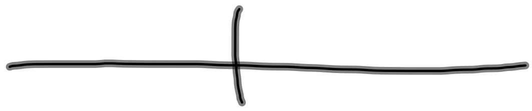
So when I take $\ln(e^x)$, I have an increasing function. More importantly, it respects the inequality.

Aug 26-10:52 AM

$e^x < 10$

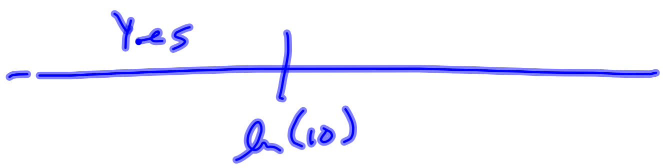
$e^x = 10$

$x = \ln(10)$

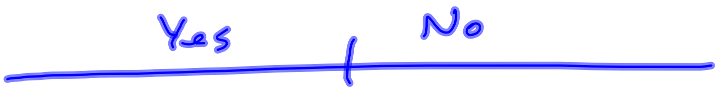


$\ln(10)$ Test e^x

$(-\infty, \ln(10))$: 0 $e^0 = 1 < 10$? Yes



$(\ln(10), \infty)$: 10 $e^{10} < 10$? No



$\ln 10$

Conclude:

$x \in (-\infty, \ln(10))$

Aug 30-12:16 PM

43. If a bacteria population starts with 100 bacteria and doubles every three hours, then the number of bacteria after t hours is $n = f(t) = 100 \cdot 2^{t/3}$.
(a) Find the inverse of this function and explain its meaning.
(b) When will the population reach 50,000?

$f(t) = y = 100 \cdot 2^{\frac{t}{3}}$

$t = 100 \cdot 2^{\frac{y}{3}} = t$

$2^{\frac{y}{3}} = \frac{1}{100} t$

$\log_2 (2^{\frac{y}{3}}) = \log_2 (\frac{1}{100} t)$

$\frac{y}{3} = \log_2 (\frac{1}{100} t)$

$y = 3 \log_2 (\frac{t}{100}) = f^{-1}(t)$

I don't like this notation for this PHYSICAL SITUATION -

I've lost sight of the physical.
Better to leave t's & y's as they are.
Then $t = 3 \log_2 (\frac{y}{100})$ is the inverse function,
where t = time
y = Population of the bacteria.
So the inverse gives time as a function of population.

(b) $t(y) = 3 \log_2 (\frac{y}{100})$
$t(50{,}000) = 3 \log_2 (\frac{50000}{100}) = 3 \log_2 (500) = 3 \frac{\ln(500)}{\ln(2)}$
≈ 26.897

$(ab)^c = a^c + b^c \qquad \ln(ab) = \ln a + \ln b$

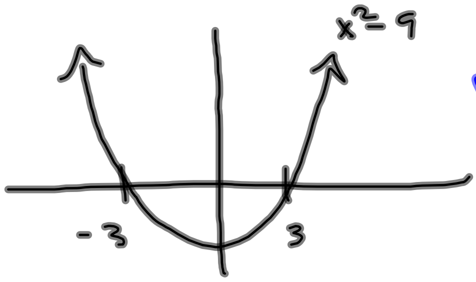
45–50 Find the limit.

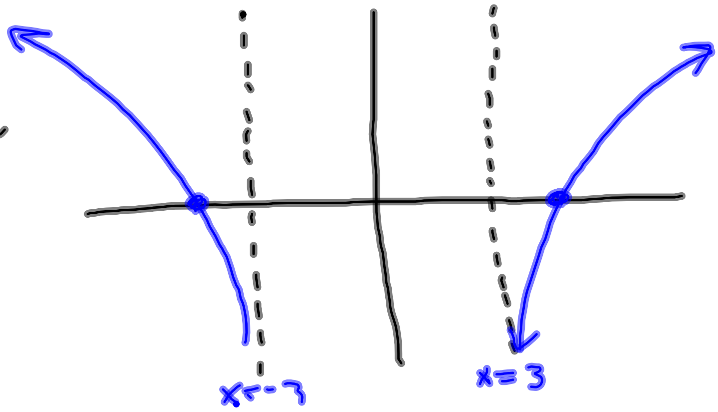
45. $\lim\limits_{x\to 3^+} \ln(x^2-9) = \lim\limits_{x\to 3^+}\left[\ln\big((x-3)(x+3)\big)\right] =$

$$\underbrace{\lim_{x\to 3^+}\left[\ln(x-3) + \ln(x+3)\right] = \lim_{x\to 3^+}\underbrace{\big(\ln(x-3)\big)}_{-\infty} + \lim_{x\to 3^+}\underbrace{\big(\ln(x+3)\big)}_{\ln(6)}}_{} = -\infty$$

$\lim (f+g) = \lim f + \lim g$, provided $\lim f$ & $\lim g$ both exist and are finite. Teacher's playing fast & loose with limit property.

what's $\ln(x^2-9)$ look like?





$\lim\limits_{x\to 3^+}\left(\ln(x^2-9)\right) = -\infty$

49. $\lim_{x\to\infty} [\ln(1+x^2) - \ln(1+x)]$

Grows without bound

Grows without bound.

$$= \lim_{x\to\infty}\left(\ln\left(\frac{x^2+1}{x+1}\right)\right) = \ln\left(\lim_{x\to\infty}\left(\frac{x^2+1}{x+1}\right)\right),$$ provided it exists.

But $\frac{x^2+1}{x+1} = \frac{x^2(1+\frac{1}{x^2})}{x^2(\frac{1}{x}+\frac{1}{x^2})} = \frac{1+\frac{1}{x^2}}{\frac{1}{x}+\frac{1}{x^2}} \xrightarrow{x\to\infty} \infty$

$\frac{1}{x^2} \to 0$, $\frac{1}{x} \to 0$, $\frac{1}{x^2} \to 0$

So we're asking

$\lim_{u\to\infty} \ln(u) = \infty.$

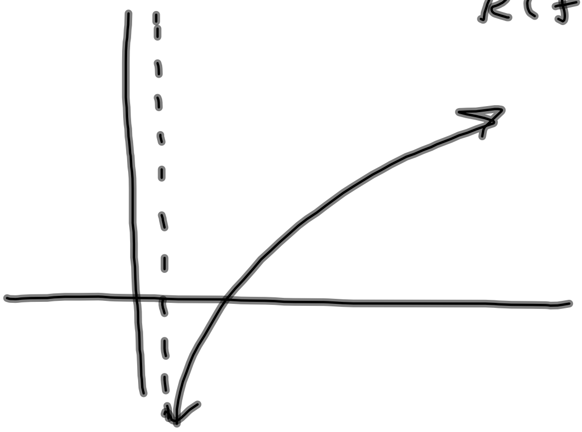
#54

$f(x) = \ln(2 + \ln x)$

$\ln x > -2$

$x > e^{-2} = \mathcal{D}(f) = \mathcal{R}(f^{-1}) = (e^{-2}, \infty)$

$\mathcal{R}(f) = (-\infty, \infty) = \mathcal{D}(f^{-1})$



$e^{-2} = x$

Aug 29-1:04 PM

53–54 Find (a) the domain of f and (b) f^{-1} and its domain.

53. $f(x) = \sqrt{3 - e^{2x}}$

$\mathcal{D} = \{x \mid f(x) \text{ is real}\}$

$= \{x \mid 3 - e^{2x} \geq 0\}$

$-e^{2x} + 3 \geq 0$

$-e^{2x} \geq -3$

$e^{2x} \leq 3$

$\boxed{\ln(e^{2x}) \leq \ln(3)}$

Reverse the sense of the inequality.

Relational operators: $=, \neq, <, >, \leq, \geq$

ln(*) is an increasing function of its argument. Its argument, e^{2x} is increasing

$\mathcal{R}(e^{2x})$

$(0, \infty)$ as x increases, e^{2x} increases.

$f(x) = -x$ is decreasing. As x increases, $f(x)$ shrinks.

So what does $f(e^{2x})$ do?

It's decreasing.

$\boxed{f(e^{2x}) = -e^{2x}}$ as $x \to$ Bigger, $e^{2x} \to$ Bigger, $f(e^{2x})$ shrinks, is decreasing.

61. On what interval is the function $f(x) = e^{3x} - e^{x}$ increasing?

$f'(x) = 3e^{3x} - e^{x} \overset{\text{SET}}{=} 0$

$3x = 2x + x$

$e^{3x} = e^{2x+x} = e^{2x}e^{x}$

$e^{x}(3e^{2x} - 1) = 0$

$3e^{2x} = 1$

$e^{2x} = \frac{1}{3}$

$2x = \ln(1/3)$

$x = \frac{1}{2}\ln(\frac{1}{3})$

$-$ $+$ f'

$\frac{1}{2}\ln(\frac{1}{3})$

Increasing on $(-\frac{1}{2}\ln(3), \infty)$

I need to refresh on whether we include the endpoint

Attachments

5-1-spread.xlsx

cosine-animation-riemann.wmf

5-1-spread-for-lecture.xlsx