EXAMPLE

Find the inverse function for $f(x) = x^2 + 6x - 7$.

$(x+3)^2 - 16$

Find the inverse function for $f(x) = x^2 + 6x - 7$ (Restrict the domain to $\{x \mid x \le -3\}$

$y^2 + 6y - 7 = x$

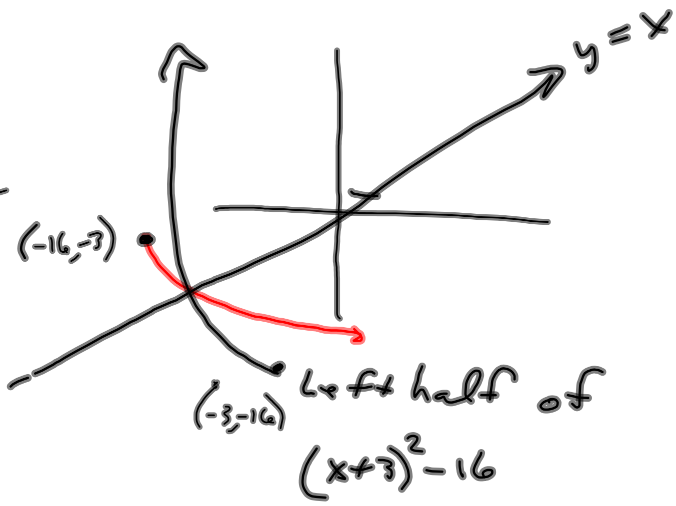
$y^2 + 6y = x + 7$

$y^2 + 6y + 3^2 = x + 7 + 3^2$

$(y+3)^2 = x + 16$

$y + 3 = \pm\sqrt{x+16}$

$y = -3 \pm \sqrt{x+16}$



7 THEOREM If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

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(a) Show that f is one-to-one.

(b) Use Theorem 7 to find $(f^{-1})'(a)$.

(c) Calculate $f^{-1}(x)$ and state the domain and range of f^{-1}.

(d) Calculate $(f^{-1})'(a)$ from the formula in part (c) and check that it agrees with the result of part (b).

(e) Sketch the graphs of f and f^{-1} on the same axes.

$f(x) = x^2+6x-7,\ x \geq -3$

$f^{-1}(x) = \sqrt{x+16} - 3$

$f'(x) = 2x+6$

$f^{-1}(0) = \sqrt{16} - 3$
$= 4-3$
$= 1$

$f'(1) = 2(1)+6 = 2+6 = 8$

$$(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(1)} = \frac{1}{8}$$

$f^{-1}(x) = (x+16)^{\frac{1}{2}} - 3 \implies$

$$(f^{-1})'(x) = \frac{1}{2}(x+16)^{-\frac{1}{2}}$$

$$(f^{-1})'(0) = \frac{1}{2}(0+16)^{-\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{16}}\right) = \frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{8}$$

5-1-spread.xlsx

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5-1-spread-for-lecture.xlsx