$F(y) = y \ln(1+e^{y})$

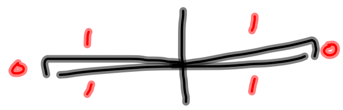
§7.1 #14 Is it 1-to-1?

$h(x) = 1 + \cos(x) \quad 0 \le x \le \pi$

Kelly's idea!

Find zeros of $h'(x)$. If none fall in the interior, $(0, \pi)$, then done. Yes, 1-to-1.

Nice idea



$h'(x) = -\sin(x) \overset{SET}{=} 0$

$\Rightarrow x = 0 + 2n\pi,\ n \in \mathbb{Z}$

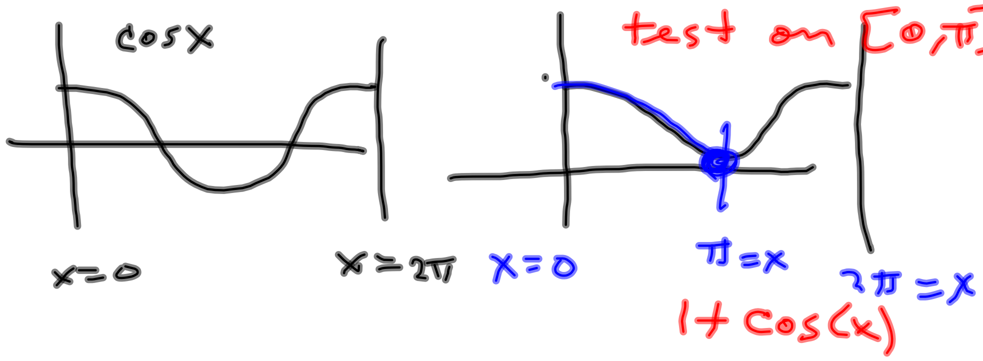
or

$x = \pi + 2n\pi,\ n \in \mathbb{Z}$

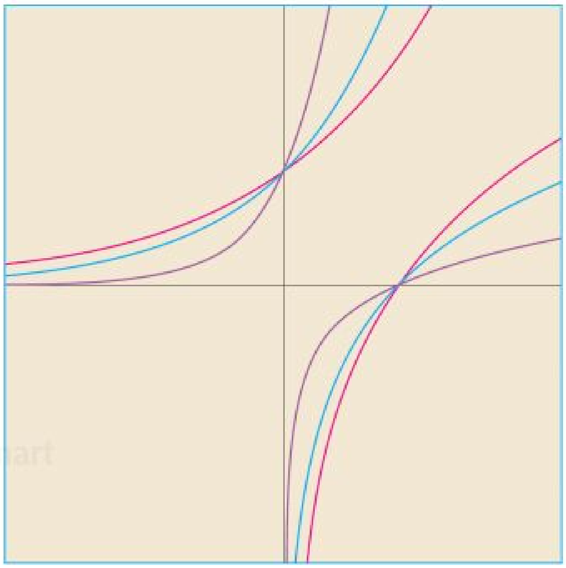
$\therefore h'(x) \ne 0$ in interior, i.e., $h'(x) > 0$ and so it's 1-to-1 on $[0, \pi]$

Steve's idea

Passes horizontal line test on $[0, \pi]$.



7 INVERSE FUNCTIONS: EXPONENTIAL, LOGARITHMIC, AND INVERSE TRIGONOMETRIC FUNCTIONS



We'll approach this in the traditional way, starting with exponential functions, and defining logarithmic functions from them.

7.1 #s 7, 17, 31, 36, 51, 59, 69, 73, 75, 92

7.1 INVERSE FUNCTIONS

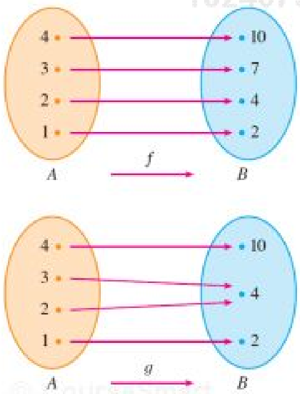


FIGURE 1

f is one-to-one; g is not

You can always find some sort of inverse RELATION, but in order for a function to have an inverse FUNCTION (a special relation), the original function must be one-to-one. So, we spend some time on these.

> **1 DEFINITION** A function f is called a **one-to-one function** if it never takes on the same value twice; that is,
>
> $$f(x_1) \neq f(x_2) \qquad \text{whenever } x_1 \neq x_2$$

1 **DEFINITION** A function f is called a **one-to-one function** if it never takes on the same value twice; that is,

$$f(x_1) \neq f(x_2) \qquad \text{whenever } x_1 \neq x_2$$

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HORIZONTAL LINE TEST A function is one-to-one if and only if no horizontal line intersects its graph more than once.

In the language of inputs and outputs, this definition says that f is one-to-one if each output corresponds to only one input.

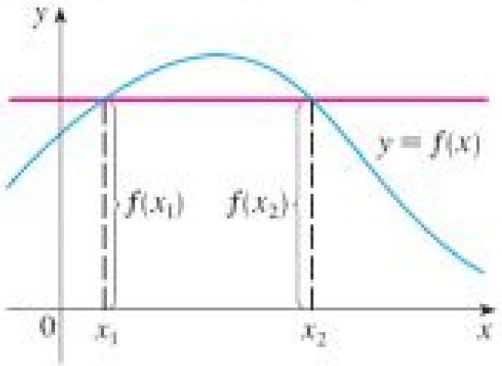

FIGURE 2
This function is not one-to-one because $f(x_1) = f(x_2)$.

EXAMPLE: Is $f(x) = x^2 + 6x - 7$ one-to-one?

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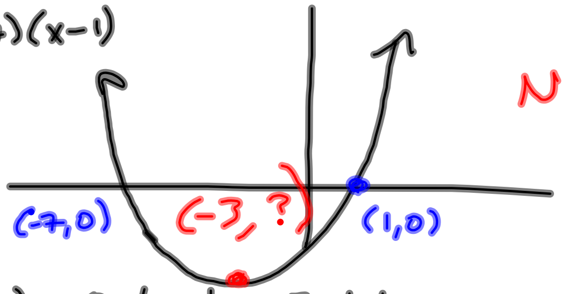
Is $f(x) = x^2 + 6x - 7$ one-to-one?

$= (x+7)(x-1)$



Not 1-to-1 by horizontal line test

$f(-7) = f(1) = 0$, but $-7 \neq 1$.

$x_2^2 + 6x_2 - 7 = x_1^2 + 6x_1 - 7$

$x_2^2 + 6x_2 + 3^2 - 7 = x_1^2 + 6x_1 + 3^2 - 7$

$(x_2+3)^2 = (x_1+3)^2$

Solving for x_2:

$\sqrt{(x_2+3)^2} = \sqrt{(x_1+3)^2}$

$|x_2+3| = |x_1+3|$

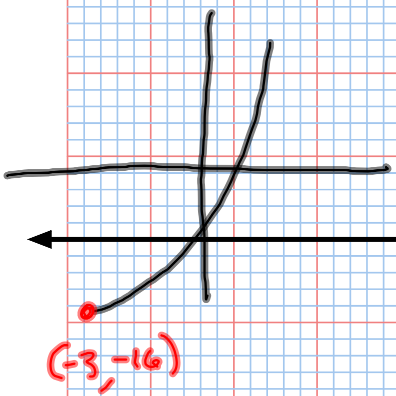
Excellent algebra stuff.

$x_2 + 3 = x_1 + 3$ OR $x_2 + 3 = -(x_1+3)$

$x_2 = x_1$ OR $x_2 = -x_1 - 6$

Two possibilities. Not 1-to-1.

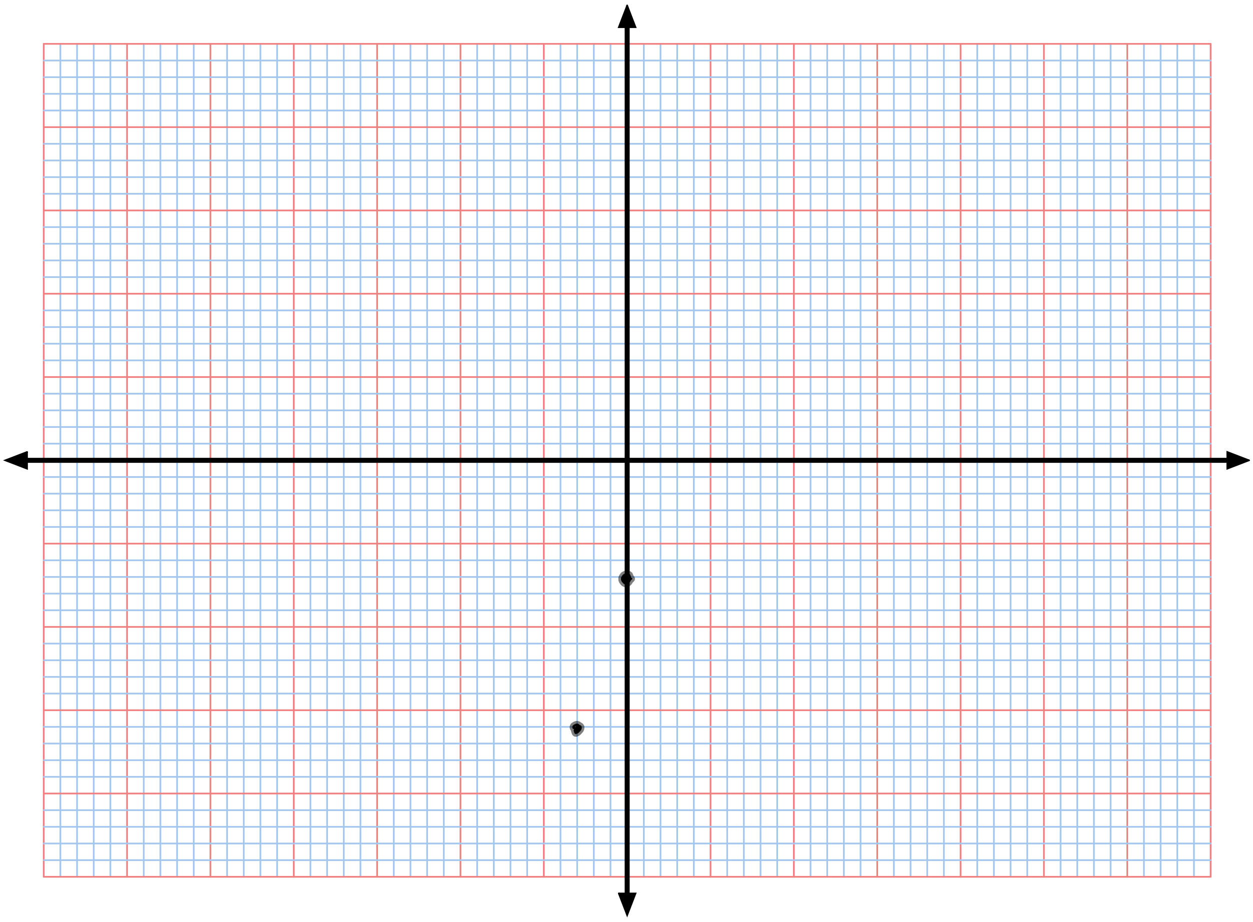
If we restrict the domain to $\{x \mid x \geq -3\}$, then x^2+6x-7 is 1-to-1 on this domain.



x^2+6x-7

$$h(-3) = (-3)^2 + 6(-3) - 7$$
$$= 9 - 18 - 7 = -16$$

$$\begin{array}{r|rrr} -3 & 1 & 6 & -7 \\ & & -3 & -9 \\ \hline & 1 & 3 & -16 \end{array}$$ ✓



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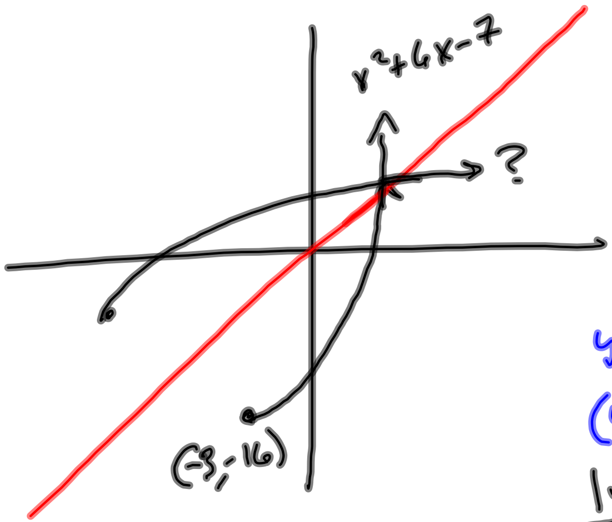


$y = x^2 + 6x - 7$

$x = y^2 + 6y - 7$

$y^2 + 6y - 7 = x$

$y^2 + 6y + 3^2 = x + 7 + 9$

$(y+3)^2 = x + 16$

$|y+3| = \sqrt{x+16}$

$y+3 = \sqrt{x+16}$ or $y+3 = -\sqrt{x+16}$

$y+3 = \pm\sqrt{x+16}$

$y = \pm\sqrt{x+16} - 3$

& $y = +\sqrt{x+16} - 3$ is the inverse of f(x) on the restricted domain.

2 DEFINITION Let f be a one-to-one function with domain A and range B. Then its **inverse function** f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \iff f(x) = y$$

for any y in B. $f^{-1}(f(x)) = x$

domain of f^{-1} = range of f

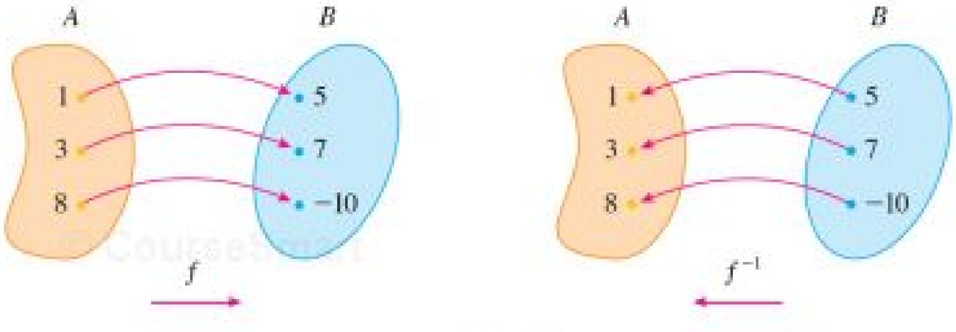
range of f^{-1} = domain of f

f^{-1} is NOT $\frac{1}{f}$ → Arithmetic multiplication.

It's the inverse with respect to the operation of function composition.

CAUTION Do not mistake the -1 in f^{-1} for an exponent. Thus

$$f^{-1}(x) \quad \text{does } \textit{not} \text{ mean} \quad \frac{1}{f(x)}$$



3

$$f^{-1}(x) = y \iff f(y) = x$$

By substituting for y in Definition 2 and substituting for x in (3), we get the following **cancellation equations**:

4

$$f^{-1}(f(x)) = x \quad \text{for every } x \text{ in } A$$
$$f(f^{-1}(x)) = x \quad \text{for every } x \text{ in } B$$

5 **HOW TO FIND THE INVERSE FUNCTION OF A ONE-TO-ONE FUNCTION f**

STEP 1 Write $y = f(x)$.

STEP 2 Solve this equation for x in terms of y (if possible).

STEP 3 To express f^{-1} as a function of x, interchange x and y. The resulting equation is $y = f^{-1}(x)$.

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EXAMPLE

Find the inverse function for $f(x) = x^2 + 6x - 7$.

Find the inverse function for $f(x) = x^2 + 6x - 7$ (Restrict the domain to $\{x \mid x \le -3\}$

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6 THEOREM If f is a one-to-one continuous function defined on an interval, then its inverse function f^{-1} is also continuous.

If time, discuss the geometric argument made in the discussion prior to Theorem 7:

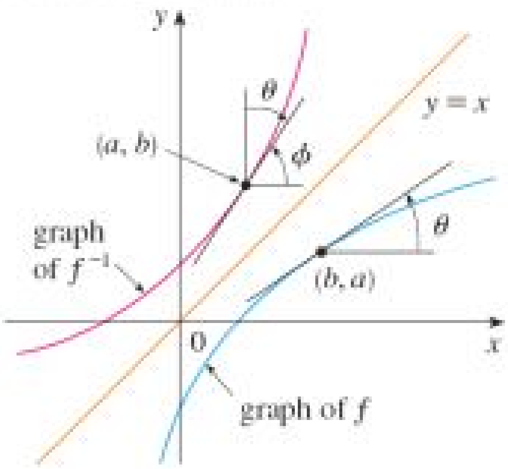

FIGURE 11

$$(f^{-1})'(a) = \tan\phi = \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta = \frac{1}{\tan\theta} = \frac{1}{f'(b)}$$

7 THEOREM If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

33–36

(a) Show that f is one-to-one.
(b) Use Theorem 7 to find $(f^{-1})'(a)$.
(c) Calculate $f^{-1}(x)$ and state the domain and range of f^{-1}.
(d) Calculate $(f^{-1})'(a)$ from the formula in part (c) and check that it agrees with the result of part (b).
(e) Sketch the graphs of f and f^{-1} on the same axes.

$f(x) = x^2 + 6x - 7,\ x \geq -3$

$f^{-1}(x) = \sqrt{x+16} - 3$

$f'(x) = 2x + 6$

$f^{-1}(0) = \sqrt{16} - 3$
$= 4 - 3$
$= 1$

$f'(1) = 2(1) + 6 = 2 + 6 = 8$

$$(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(1)} = \frac{1}{8}$$

$f^{-1}(x) = (x+16)^{\frac{1}{2}} - 3 \implies$

$(f^{-1})'(x) = \frac{1}{2}(x+16)^{-\frac{1}{2}}$

$(f^{-1})'(0) = \frac{1}{2}(0+16)^{-\frac{1}{2}}$

$= \frac{1}{2}\left(\frac{1}{\sqrt{16}}\right) = \frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{8}$

Attachments

5-1-spread.xlsx

cosine-animation-riemann.wmf

5-1-spread-for-lecture.xlsx