MAT 202 Final Cheat Sheet

$\binom{k}{0} = 1$, $\binom{k}{1} = k$, $\binom{k}{2} = \frac{k\cdot(k-1)}{2!}$, $\binom{k}{3} = \frac{k\cdot(k-1)\cdot(k-2)}{3!}$

$\binom{k}{4} = \frac{k\cdot(k-1)\cdot(k-2)\cdot(k-3)}{4!}$,......, $\binom{k}{n} = \frac{k\cdot(k-1)\cdot(k-2)\cdots(k-n+1)}{n!}$

$$(1+x)^k = \sum_{n=0}^{\infty}\binom{k}{n}x^n$$

In the case k is a positive integer, this series terminates at $n = k$, and takes the form of our old, familiar College Algebra Binomial Theorem, and the Binomial coefficients, can be found using Pascal's triangle. This powerful result gives a way to handle power series representations for functions like

$$f(x) = \frac{1}{\sqrt{a+x}} = \frac{1}{\sqrt{a}}\cdot\frac{1}{\sqrt{1+\frac{x}{a}}}$$

$$\frac{d}{dx}[arctan(x)] = \frac{1}{1+x^2}$$

Trig Substitution: $\sqrt{a^2 - x^2} \Rightarrow x = a\cdot sin(\theta)$, $\sqrt{a^2+x^2} \Rightarrow x = a\cdot tan(\theta)$,

$\sqrt{x^2 - a^2} \Rightarrow x = a\cdot sec(\theta)$

Partial fractions: If the rational function is *improper*, use long division. This will yield a polynomial plus a rational function, where the rational function is *proper*. To break down a *proper* rational function, we solve the equation

$$\frac{a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0}{\left(b_2x^2 + b_1x + b_0\right)(x-d)^3} = \frac{Ax+B}{\left(b_2x^2+b_1x+b_0\right)} + \frac{C}{x-d} + \frac{D}{(x-d)^2} + \frac{E}{(x-d)^3}.$$

Area in polar coordinates: $r = f(\theta) \rightarrow A = \frac{1}{2}\int_a^b f(\theta)^2\, d\theta$

Arc Length, when $x = f(t),\ y = g(t)$ is given by $S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Taylor Series: $f(x) = \sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$

Taylor Inequality: $|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1}$. In general $|x-a|$ is the maximum distance d from x to a on the interval in question, typically the radius of the interval, centered at a. For a particular value of x, it's just the distance from x to a. M is the maximum of the $(n+1)^{th}$ derivative on the interval, and the interval is either given, outright, or the interval between a and the x-value we're plugging into $f(x)$.

Arc Length: $y = f(x) \Rightarrow S = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$. Swap x and y when $x = g(y)$.

Area of surface of revolution: $A = 2\pi\int_a^b y\, ds$ when we rotate about the x-axis.

The ds comes from Arc Length.