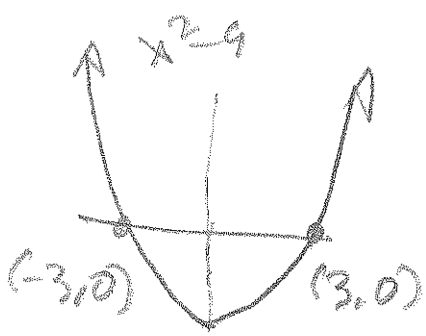
202 §6.3 IV #s 47, 51, 53, 55, 59, 64–66, 68, (72)

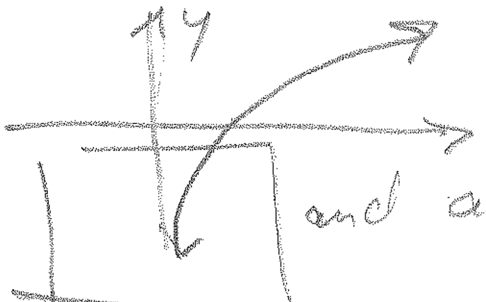
#s 47–52 Find the limit.

47 $\lim_{x \to 3^+} \ln(x^2-9)$. We must analyze x^2-9, because its OUTPUTS are the INPUTS to the natural log function, outside.



$\mathcal{D}(\ln(x^2-9)) = (-\infty,-3) \cup (3,\infty)$, because that's where $x^2-9>0$.

So, ~~coming into~~ approaching $x=3$ from the right, you see that $x^2-9 \xrightarrow{x \to 3^+} 0$, and so we're looking at this piece of $\ln(*)$ function



and as $* \to 0^+$, $\ln(*) \longrightarrow -\infty$!

$\therefore$ $\lim_{x \to 3^+} \ln(x^2-9) = \lim_{* \to 0^+} \ln(*) \boxed{= -\infty}$

§6.3 #s 51, 53, 55, 58, 59, 64, 66, 68, (72)

#s 47-52 Find the limit

(51) $\lim\limits_{x \to \infty} \left[\ln(1+x^2) - \ln(1+x) \right]$

$= \infty - \infty$ is unclear. By combining

$= \lim\limits_{x \to \infty} \ln\left(\dfrac{x^2+1}{x+1}\right) = \ln\left(\lim\limits_{x \to \infty} \left(\dfrac{x^2+1}{x+1}\right)\right)^{*}$

$= \boxed{\infty}$, since $\dfrac{x^2+1}{x+1} \xrightarrow{x \to \infty} \infty$

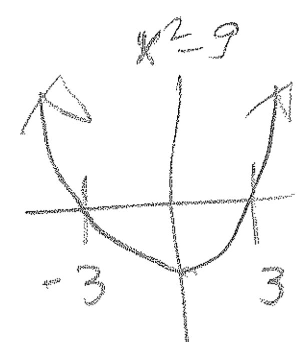
and $\ln(\square) \xrightarrow{\square \to \infty} \infty$

Combining into one log can help, but it can also change the domain on you.

#s 53-4 Find domain of the function.

(53) $f(x) = \log(x^2-9) \Longrightarrow \mathcal{D}(f) = \{x \mid x^2-9>0\}$

$= \boxed{(-\infty,-3) \cup (3,\infty) = \mathcal{D}(f)}$



202 §6.3 #s 55, 58, 59, 64, 66, 68, (72)

#s 55-57

(a) Find $\mathcal{D}(f)$

(b) Find f^{-1} & $\mathcal{D}(f^{-1})$

55 $f(x)=\sqrt{3-e^{2x}}$

$\boxed{\mathcal{D}(f)=\left(-\infty,\frac{\ln 3}{2}\right)}$

a Need $3-e^{2x}\ge 0$

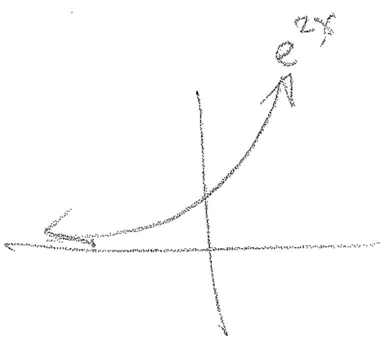
i.e., $e^{2x}-3\le 0$

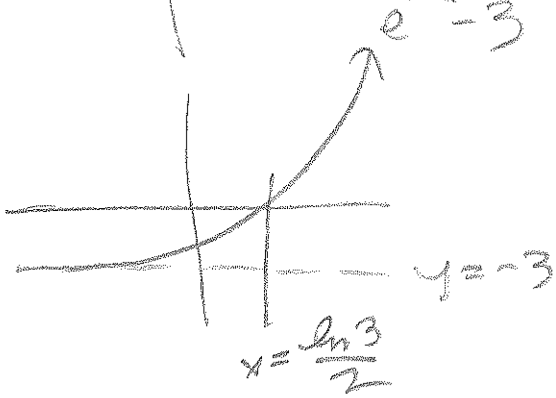
Solve $e^{2x}-3=0$

$e^{2x}=3$

$2x=\ln 3$

$x=\frac{\ln 3}{2}$





See?

Want $e^{2x}-3\le 0$

So $x\le\frac{\ln 3}{2}$

202 §6.3 #s 55, 58, 59, 64, 66, 68, (72)

55 (b) $f(x) = \sqrt{3 - e^{2x}} = y$

Switch $\sqrt{3 - e^{2y}} = x$

Solve for y: $3 - e^{2y} = x^2$

$-e^{2y} = x^2 - 3$

$e^{2y} = 3 - x^2$

$2y = \ln(3 - x^2)$

$y = \frac{1}{2}\ln(3 - x^2)$

$= \ln\sqrt{3 - x^2}$

if you want to re-write it.

$\mathcal{D}(f^{-1})$:

$3 - x^2 > 0$

$x > 3$ OR $x < -3$

$\boxed{(-\infty, -3) \cup (3, \infty)}$

What's wrong with this?

58 The point is that

$e^{\ln(300)} = \ln(e^{300}) = 300$, but your calculator will choke on e^{300} before it evaluates $\ln(e^{300})$.

#s 59-64 Find the inverse function

202 §6.3 II #s 59, 64, 66, 68 (72)

59

$$y = \ln(x+3)$$

$$\ln(y+3) = x$$

$$y+3 = e^{x}$$

$$y = \boxed{e^{x} - 3 = f^{-1}(x)}$$

64

$$y = \frac{e^{x}}{2e^{x}+1}$$

$$\frac{e^{y}}{2e^{y}+1} = x \quad \text{Cross-multiply. Distribute.}$$

$$e^{y} = 2xe^{y} + x$$

$$e^{y} - 2xe^{y} = x$$

$$e^{y}(1-2x) = x$$

$$e^{y} = \frac{x}{1-2x}$$

$$y = \boxed{\ln\left(\frac{x}{1-2x}\right) = f^{-1}(x)}$$

202 §6.3 II #s 66, 68, (72)

(66) On what interval is $y = 2e^{x} - e^{-3x}$ concave downward?

Want $y'' < 0$?

$y' = 2e^{x} + 3e^{-3x}$

$y'' = 2e^{x} - 9e^{-3x} \overset{SET}{=} 0 \implies$

$e^{x}\left[2 - 9e^{-4x}\right] = 0 \implies$

$-9e^{-4x} + 2 = 0 \implies$

$9e^{-4x} = 2$

$e^{-4x} = \frac{2}{9}$

$-4x = \ln(2/9)$

$x = -\frac{\ln(2/9)}{4} = \frac{\ln(9/2)}{4}$

$$\boxed{y'' < 0 \text{ when } x < \frac{\ln(9/2)}{4}}$$

$-$ $\quad$ $+$

$\frac{\ln(9/2)}{4}$

For sign pattern, note that

$-9e^{-3x} + 2e^{x} = e^{x}\left[-9e^{-4x} + 2\right]$

is negative when $|-9e^{-4x}|$ is big.

That happens to the left.

Calculator can help too, with test values

202 § 6.3 II #s 68, (72)

(68) Find eq'n of tan line to $y = e^{-x}$ that is perpendicular to $2x - y = 8$

Recall $m_{\perp} = -\frac{1}{m}$

$2x - y = 8 \Longrightarrow m = 2 \Longrightarrow \boxed{m_{\perp} = -\frac{1}{2}}$

$y' = -e^{-x} \stackrel{\text{set}}{=} -\frac{1}{2}$

$\Longrightarrow e^{-x} = \frac{1}{2}$

$-x = \ln(1/2)$

$x = -\ln(1/2) = \ln(2)$

$y\Big|_{x=\ln 2} = e^{-\ln(2)} = e^{\ln(1/2)} = \frac{1}{2}$

So $(x_1, y_1) = (\ln 2, \frac{1}{2})$ &

tangent line is

$$\boxed{y = -\frac{1}{2}(x - \ln 2) + \frac{1}{2}}$$