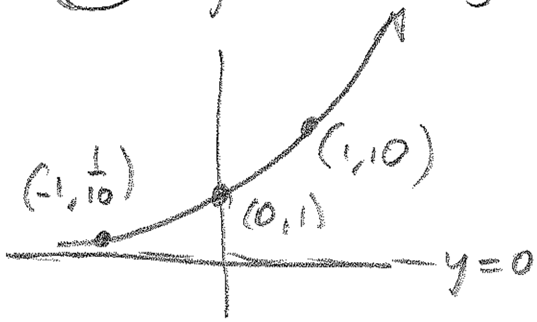
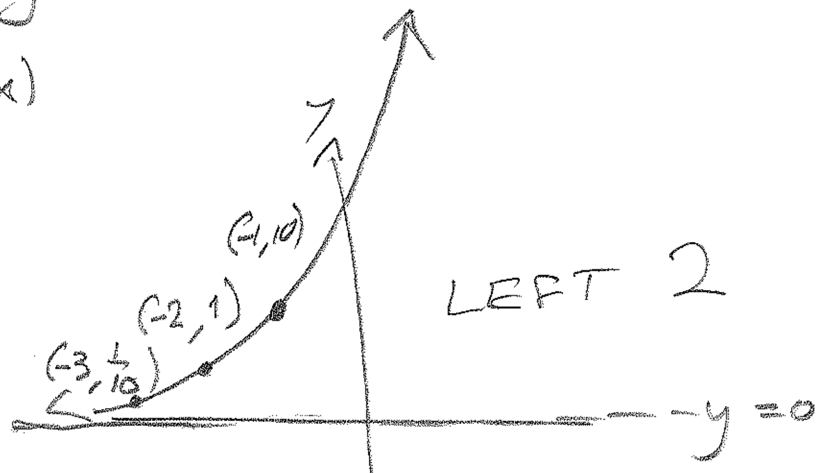
202 § 6.2 #s 7-9, 15-55 odds

#s 7-12 make a rough sketch.

(7) $y = 10^{x+2} = g(x)$



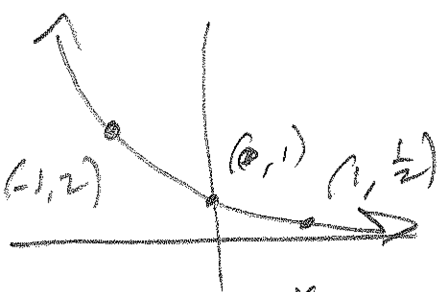
$y = 10^x = f(x)$



LEFT 2

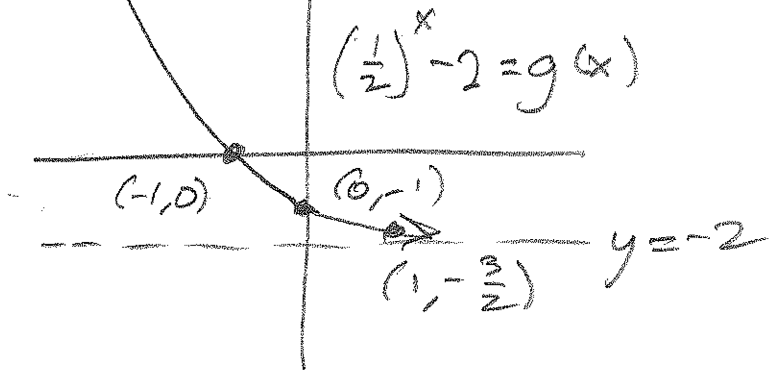
$f(x+2) = 10^{x+2} = g(x)$

(8) $y = (0.5)^x - 2 = \left(\frac{1}{2}\right)^x - 2 = g(x)$

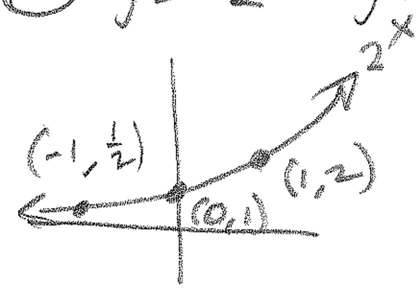


$y = \left(\frac{1}{2}\right)^x = f(x)$

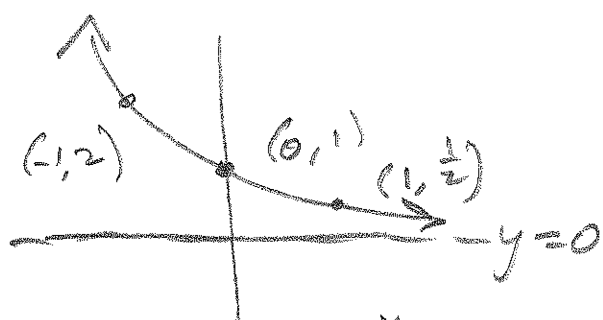
Down 2



(9) $y = -2^{-x} = g(x)$

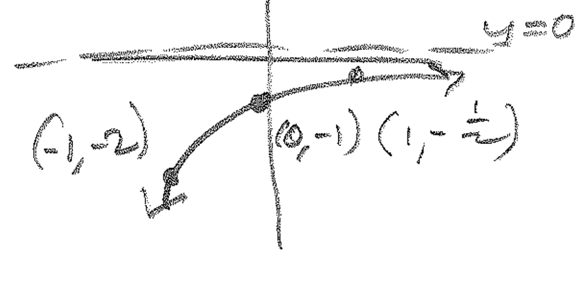


$f(x) = 2^x$



$f(-x) = 2^{-x}$

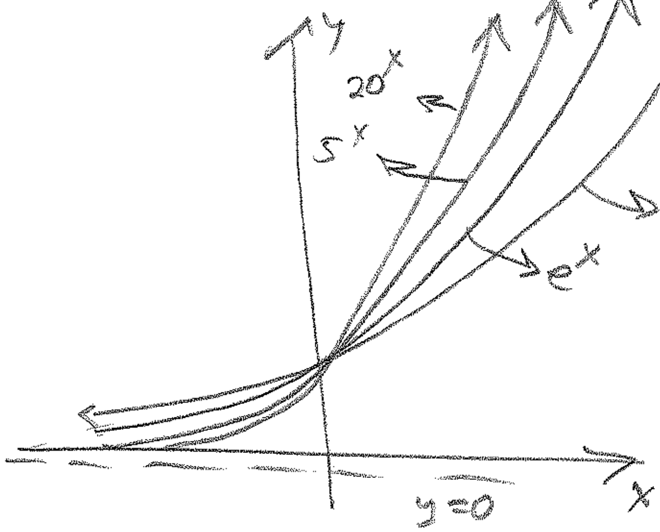
$g(x) = -2^{-x}$
$= -f(-x)$



202 § 6.2 #s 3, 5, 7–9, 15–55 odds

#s 3–6 Graph on common screen. How are the graphs related?

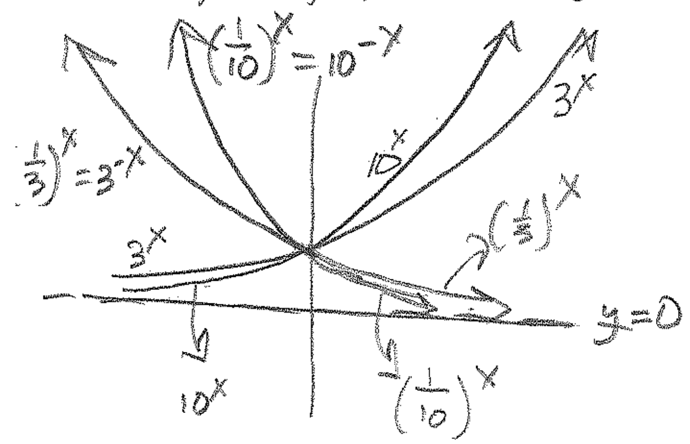
3. $y = 2^x,\ y = e^x,\ y = 5^x,\ y = 20^x$



For $x > 0$,
$2^x < e^x < 5^x < 20^x$
For $x < 0$
$2^x > e^x > 5^x > 20^x$

All are concave up.
All satisfy
$\lim\limits_{x \to -\infty} f(x) = 0$ & $f(0) = 1$
$\lim\limits_{x \to \infty} f(x) = \infty$
$f(x) > 0 \ \forall\ x$

5. $y = 3^x,\ y = 10^x,\ y = \left(\frac{1}{3}\right)^x,\ y = \left(\frac{1}{10}\right)^x$



$10^{-x} = \left(\frac{1}{10}\right)^x$ is reflection of 10^x thru y-axis.

$3^{-x} = \left(\frac{1}{3}\right)^x$ is reflection of 3^x thru y-axis.

202 § 6.2 15–55 odds

(15) Find $\mathcal{D}(f)$ for $f(x) = \dfrac{1-e^{x^2}}{1-e^{1-x^2}}$

Need $1-e^{1-x^2} \neq 0$

$e^{1-x^2} = 1$ $\quad e^{\square} = 1$ iff $\square = 0$

$1-x^2 = 0$

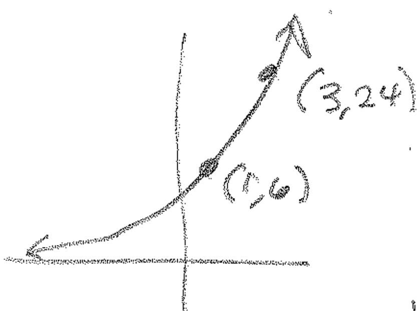
$(1-x)(1+x) = 0$

$x \in \{-1, 1\} \longrightarrow$

$\mathcal{D}(f) = \mathbb{R} \setminus \{-1,1\} = \{x \mid x \neq \pm 1\} =$

$= (-\infty,-1) \cup (-1,1) \cup (1,\infty)$

(17) Use the graph to determine the function Ca^x



$\boxed{M1}$ $Ca^1 = 6$ $\quad Ca^3 = 24$ $\implies$ $Ca = 6$, $\quad a = \dfrac{6}{C}$

$\implies C\left(\dfrac{6}{C}\right)^3 = 24$

$C\dfrac{6^3}{C^3} = 24$

$\dfrac{6^3}{C^2} = 6 \cdot 4$

$6^2 = 4C^2$

$9 = C^2$

$\pm 3 = C \implies \boxed{C = +3} \rightarrow 3a = 6$

$\boxed{a = 2}$

$\boxed{Ca^x = 3 \cdot 2^x}$

$\boxed{M2}$

$Ca - Ca^3 = 6 - 24$

$Ca - Ca^3 = -18$

$C(a - a^3) = -18$

Looks messy.

Bleah.

202 §6.2 19-55 odds

(19) $f(x) = x^2$ & $g(x) = 2^x$ are drawn on a coordinate grid where unit of measurement is 1 inch, i.e. x = distance, in inches. Show that, at a distance of 2 ft to the right of $x=0$, the height of f is 48 ft, but the height of g is 265 mi!

The purpose of the exercise is two-fold:

(1) Play with unit conversions

(2) Show that 2^x gets much bigger than x^2, eventually, and pretty quickly.

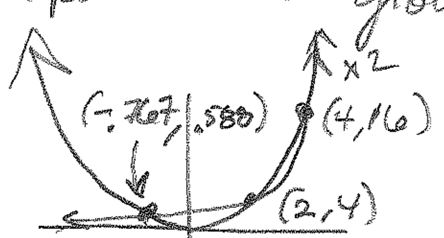
$$x = (2\,ft)\left(\frac{12\text{ in}}{1\text{ ft}}\right) = 24\text{ in}$$

$$f(24) = 24^2 = 576\text{ in} = (576\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right) = 48\text{ ft}$$

$$g(24) = 2^{24} = (16777216\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)\left(\frac{1\text{ mi}}{5280\text{ ft}}\right)$$

$$\approx 264.7919192\text{ mi} \approx 265\text{ mi}$$

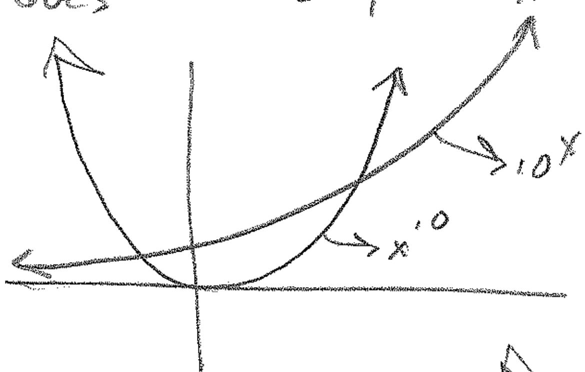
Exponentials grow way faster than polynomials!



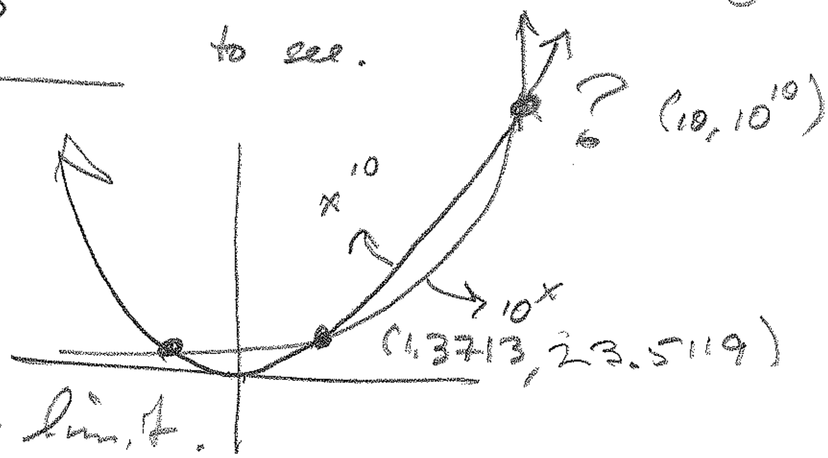
x^2 & 2^x intersect 3 places – It takes tech to get one of them. Guessing can get you the other two, but there's no algebraic method for $2^x = x^2$

202 §6.2 #s 21-55 odds

(21) GRAPHER! Compare x^{10} & 10^x. When does 10^x surpass x^{10}? $\boxed{\text{at } x = 10}$



Eventually 10^x catches up. But it's tough to see.



#s 23-30 Find the limit.

(23) $\lim_{x\to\infty} (1.001)^x = \infty$ (or "$\nexists$", depending on the context of the question, b/c $\infty \notin \mathbb{R}$)

(25) $\lim_{x\to\infty} \dfrac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$

$= \lim_{x\to\infty} \dfrac{e^{3x}(1 - e^{-6x})}{e^{3x}(1 + e^{-6x})}$

$\dfrac{e^{3x}}{e^{3x}} = 1$ $\quad \dfrac{e^{-3x}}{e^{3x}} = \dfrac{1}{e^{3x+3x}} = \dfrac{1}{e^{6x}} = e^{-6x}$

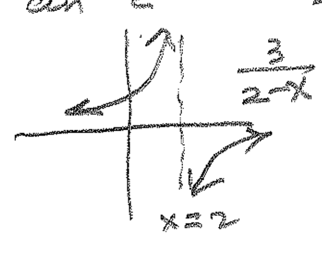
$\boxed{= 1}$

(27) $\lim_{x\to 2^+} e^{\frac{3}{2-x}}$ $\boxed{= 0}$ $\leftarrow$ ($e^{-\infty}$ "=" 0)

It's an $e^{-\text{stuff}}$ sitch.

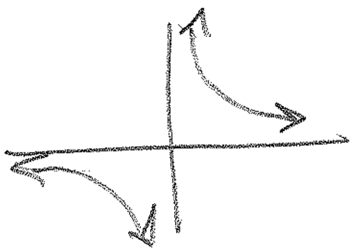
So $x > 2$ means $2 - x < 0$.

and $\dfrac{3}{2-x} \xrightarrow{x\to 2^+} -\infty$

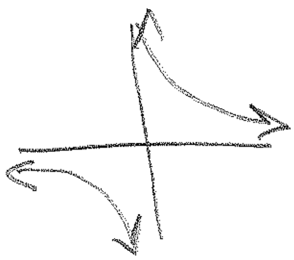


203 §6.2 #s 27-55 ODDS

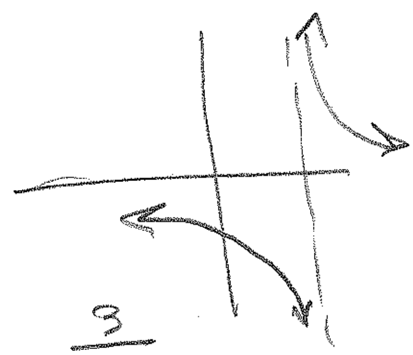
27) We analyzed $\frac{3}{2-x}$, separately, to get a handle on $e^{\frac{3}{2-x}}$



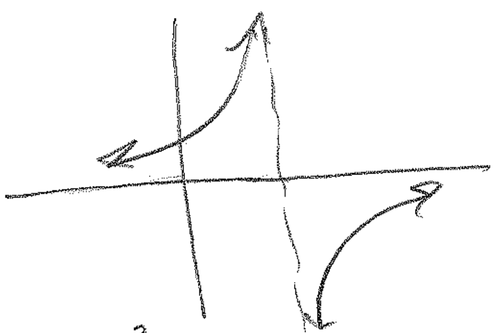
$\frac{1}{x} = f(x)$



$\frac{3}{x} = 3f(x)$



$\frac{3}{x-2}$ $= 3f(x-2)$ $x=2$



$\frac{-3}{x-2} = \frac{3}{2-x}$ $x=2$

$= -3f(x-2)$

29) $\lim\limits_{x\to\infty}\left(e^{-2x}\cos(x)\right) = 0 \cdot (\text{something between} \pm 1)$ $\boxed{= 0}$

$e^{-2x} \xrightarrow{x\to\infty} 0$

$-1 \le \cos(x) \le 1$

Squeeze!

$-1 \le \cos x \le 1$

$-e^{-2x} \le e^{-x}\cos x \le +e^{-2x}$

$\downarrow x\to\infty \qquad \downarrow x\to\infty \qquad \downarrow x\to\infty$

$0 \le \lim\limits_{x\to\infty} e^{-2x}\cos(x) \le 0$

202 §6.2 #s 31-55 odds

#s 31-50 Differentiate

(31) $f(x) = e^5 \Longrightarrow \boxed{f'(x) = 0}$

constant!

(33) $f(x) = (x^3+2x)e^x$

$\Longrightarrow \boxed{f'(x) = (3x^2+2)e^x + (x^3+2x)e^x}$

You can sorta clean this up:

$e^x(x^3+3x^2+2x+2) = f'(x)$

(35) $y = e^{ax^3} \Longrightarrow$

$\boxed{y' = 3ax^2 e^{ax^3}}$ Chain Rule

(37) $y = xe^{-kx}$ (Assuming k is konstant)

$\Longrightarrow y' = e^{-kx} - kxe^{-kx}$

Product Rule & Chain Rule

(39) $f(u) = e^{\frac{3}{u}} = e^{3u^{-1}} \Longrightarrow$

$\boxed{f'(u) = -3u^{-2}e^{\frac{3}{u}}}$

202 §6.2 #s 41–55 (ODDS)

(41) $f(t) = e^{t\sin(2t)} \Longrightarrow$

$$\boxed{f'(t) = \left(\sin(2t) + 2t\cos(2t)\right) e^{t\sin(2t)}}$$

$$\frac{d}{dt}\left[t\sin(2t)\right] = 1\cdot\sin(2t) + (t)\underbrace{\left((\cos(2t))(2)\right)}_{\text{chain on } \sin(2t)}$$

(43) $y = \sqrt{1+2e^{3x}} = \left(2e^{3x}+1\right)^{\frac{1}{2}} \Longrightarrow$

$$\boxed{y' = \frac{1}{2}\left(2e^{3x}+1\right)^{-\frac{1}{2}}\left(6e^{3x}\right)}$$

(45) $y = e^{e^x} \longrightarrow \boxed{y' = (e^x)(e^{e^x})}$

(47) $y = \dfrac{ae^x+b}{ce^x+d} \Longrightarrow$ $\qquad \left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$

$$\boxed{y' = \frac{ae^x(ce^x+d) - (ae^x+b)(ce^x)}{(ce^x+d)^2}}$$

(49) $y = \cos\left(\dfrac{1-e^{2x}}{1+e^{2x}}\right) \Longrightarrow$

$$\boxed{y' = -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\left(\frac{-2e^{2x}(1+e^{2x}) - (1-e^{2x})(2e^{2x})}{(1+e^{2x})^2}\right)}$$

202 §6.2 #s 51-55 odds

(51) Find eq'n of tangent line to $y = e^{2x}\cos(\pi x)$

(a) $(x,y) = \boxed{(x_1, y_1) = (0,1)} = (x_1, f(x_1))$

$y' = f'(x) = 2e^{2x}\cos(\pi x) - \pi e^{2x}\sin(\pi x)$

$x_1 = 0 \Longrightarrow$

$f'(x_1) = f'(0) = 2e^0\cos(0) - \pi e^0 \sin(0)$

$= 2 = m_{tan} = \boxed{f'(x_1) = 2}$

$y = m(x - x_1) + y_1$

$y = f'(x_1)(x - x_1) + f(x_1)$

$\boxed{y = 2(x-0) + 1} \longrightarrow$ Good enough for me!

$y = 2x + 1$

(53) Find y' if $e^{\frac{x}{y}} = x - y$

$\frac{y - xy'}{y^2} e^{\frac{x}{y}} = 1 - y'$

$(y - xy')e^{\frac{x}{y}} = y^2(1 - y')$

$ye^{\frac{x}{y}} - xy'e^{\frac{x}{y}} = y^2 - y^2y'$

$-xy'e^{\frac{x}{y}} + y^2y' = y^2 - ye^{\frac{x}{y}}$

$y'(y^2 - xe^{\frac{x}{y}}) = y^2 - ye^{\frac{x}{y}}$

$\boxed{y' = \frac{y^2 - ye^{\frac{x}{y}}}{y^2 - xe^{\frac{x}{y}}}}$

202 S16.2 #55

(55) Show that $y = e^x + e^{\frac{-x}{2}}$ satisfies

$2y'' - y' - y = 0$

$y' = e^x - \frac{1}{2}e^{-\frac{1}{2}x}$

$y'' = e^x + \frac{1}{4}e^{-\frac{1}{2}x}$

$$2y'' - y' - y = 2\left(e^x + \frac{1}{4}e^{-\frac{1}{2}x}\right) - \left(e^x - \frac{1}{2}e^{-\frac{1}{2}x}\right) - \left(e^x + e^{-\frac{1}{2}x}\right)$$

$$= 2e^x + \frac{1}{2}e^{-\frac{1}{2}x} - e^x + \frac{1}{2}e^{-\frac{1}{2}x} - e^x - e^{-\frac{1}{2}x} = 0 \checkmark$$

So y satisfies the differential equation.

D.E.'s Talking about derivatives to discover the functions.

If you know how something moves, all you need is one or two data points on where it was, to predict where it is, where it's going, and in what manner.