202 §6.1 #s 1, 2,

(1) (a) A 1-to-1 function is a function satisfying the property "$x_1 \neq x_2 \Longrightarrow f(x_1) \neq f(x_2)$."

(b) If f is 1-to-1, then a horizontal line will intersect f's graph at most once.

(2) (a) If f is 1-to-1, $\mathcal{D}(f) = A$ & $\mathcal{R}(f) = B$, then the inverse function, f^{-1}, has $\mathcal{D}(f^{-1}) = B$ and $\mathcal{R}(f^{-1}) = A$, and if $y = f(x)$ for $x \in A$, then $f^{-1}(y) = x$; i.e, $f^{-1}(f(x)) = x$ is the identity function

(b) Your book says solve $y = f(x)$ for x & the result $x = g(y)$ is going to by your f^{-1}. I was taught to swap x & y & solve for y.

$$y = f(x) \Longrightarrow x = f(y) \Longrightarrow \ldots g(x) = f^{-1}(x) = y,$$

if you can just isolate the y.

(c) The graph of f^{-1} is obtained from the graph of f by reflecting the graph of f about the line $y = x$.

202 §6.2

#s 3-16 Determine if f is 1-to-1

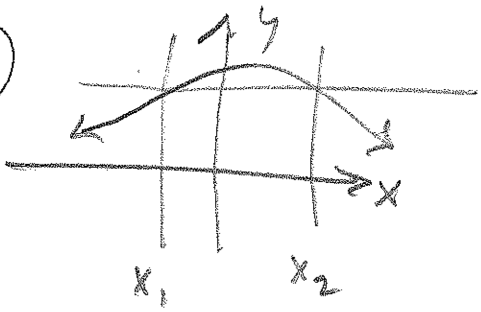
(3)

x	1	2	3	4	5	6
$f(x)$	1.5	2.0	3.6	5.3	2.8	2.0

$x_1 = 2 \neq 6 = x_2$, but

$y_1 = 2.0 = y_2 \implies$ Not 1-to-1.

(5)

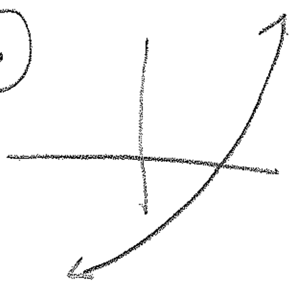


Not 1-to-1

$x_1 \neq x_2$, but $y_1 = y_2$

(Horizontal line Test

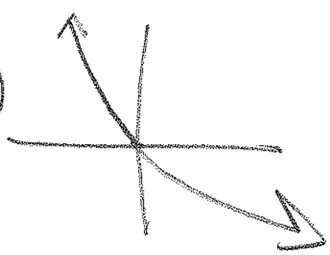
(6)



Yes. 1-to-1.

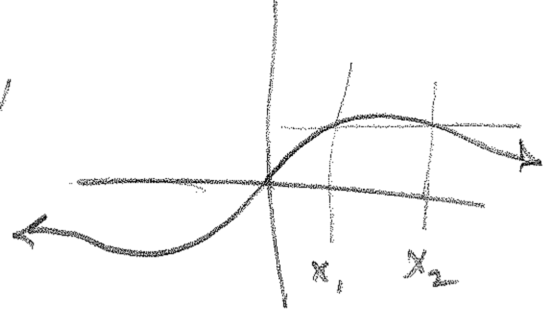
(Monotone strictly increasing)

(7)



Yes

(8) Doesn't look like it!



202 §6.1

9 $f(x) = x^2 - 2x$

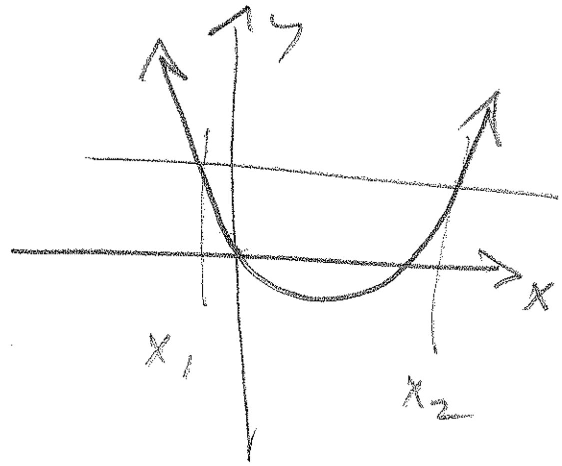
M1 $x(x-2) = 0$

$x = 0$ or $x = 2$

$\rightarrow$ $(0,0)$ & $(2,0)$

are ordered pairs of the relation. $\rightarrow$ $\boxed{\text{Not 1-to-1}}$

M2



M3 If $x_1 \neq x_2$, and f is 1-to-1. Then $y_1 \neq y_2$ ∴

$f(x_1) = x_1^2 - 2x_1 \neq x_2^2 - 2x_2$

$x_1^2 - x_2^2 - 2x_1 + 2x_2 \neq 0$

$(x_1 - x_2)(x_1 + x_2) - 2(x_1 - x_2) \neq 0$

$(x_1 - x_2)[x_1 + x_2 - 2] \neq 0$ So either

$x_1 - x_2 \neq 2$ or $x_1 + x_2 - 2 \neq 0$

$x_1 \neq 2 - x_2$. This leads to a contradiction

Let

$x_1 = 0$ & $x_2 = 2$. Then

$f(x_1) = f(x_2) = 0$.

202 §6.1

(9) (M4) Suppose $y_1 = y_2$ are in the range of f. Then $\exists$ x_1, x_2 such that

$y_1 = f(x_1)$ and $y_2 = f(x_2)$ and

$y_1 - y_2 = 0$

$x_1^2 - 2x_1 - x_2^2 + 2x_2 = 0$

$(x_1 - x_2)(x_1 + x_2 - 2) = 0$

$\underline{x_1 = x_2}$ $\underline{\text{OR}}$ $x_1 = 2 - x_2$

OK, cool. This says f isn't 1-to-1!

TO SEE THIS:

Let $x_1 = 3$ & $x_2 = 2 - 3 = -1$. Then

$$\left.\begin{aligned} f(x_1) &= 3^2 - 2(3) = 9 - 6 = 3 \\ f(x_2) &= (-1)^2 - 2(-1) = 1 + 2 = 3 \end{aligned}\right\}$$

Counter-example to claim f is 1-to-1.

So (M4) gives you a way to generate counter-examples to the claim that f is 1-to-1.

202 §6.1

11 $g(x) = \frac{1}{x}$ is 1-to-1.

Suppose $g(x_1) = g(x_2)$. Then

$$g(x_1) - g(x_2) = \frac{1}{x_1} - \frac{1}{x_2} = \frac{x_2 - x_1}{x_1 x_2} = 0 \Longrightarrow$$

$$x_2 = x_1$$

12 $g(x) = |x|$ is not 1-to-1

If $g(x_1) = g(x_2)$. Then

$$|x_1| = |x_2| \Longrightarrow$$

$x_1 = \pm x_2$ will make $g(x_1) = g(x_2)$,

and 1-to-1 requires $x_1 = x_2$ is only way this can happen.

17 $f(6) = 17 \Longrightarrow f^{-1}(17) = 6$.

19 $h(x) = x + \sqrt{x}$. Find $h^{-1}(6)$

$$x + \sqrt{x} = 6$$

$$x + \sqrt{x} - 6 = 0$$

$$(\sqrt{x}+3)(\sqrt{x}-2) = 0$$

$\cancel{\sqrt{x} = -3}$ $\quad$ $\sqrt{x} = 2$

$$x = 4$$

$$\boxed{h^{-1}(6) = 4 \ !}$$

202 Sec. 1

(22) $f(v) = m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$, where m_0 = mass at rest

c = speed of light

Find f^{-1}

$$m\left(\sqrt{1-\frac{v^2}{c^2}}\right) = m_0$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{m_0}{m}$$

$$1-\frac{v^2}{c^2} = \frac{m_0^2}{m^2}$$

$$-\frac{v^2}{c^2} = \frac{m_0^2}{m^2} - 1$$

$$v^2 = \frac{m_0^2}{m^2}c^2 - c^2 = \left(\frac{m_0^2}{m^2} - 1\right)c^2$$

$$v = \pm\sqrt{\left(\frac{m_0^2}{m^2} - 1\right)c^2} = \pm c\sqrt{\frac{m_0^2}{m^2} - 1}$$

Take the positive $v = \left(\sqrt{\frac{m_0^2}{m^2} - 1}\right)c$

202 S'b11

#s 23-28. Find inverse

23 $-2x+3 = f(x) = y$

$-2y+3 = x$

$-2y = x-3$

$y = \boxed{-\frac{1}{2}(x-3) = f^{-1}(x)}$

24 $\frac{4x-1}{2x+3} = y$

$\frac{4y-1}{2y+3} = x$

$4y-1 = x(2y+3) = 2xy+3x$

$4y-2xy = 3x+1$

$y(4-2x) = 3x+1$

$y = \boxed{\frac{3x+1}{4-2x} = f^{-1}(x)}$

28 $y = 2x^2 - 8x, \quad x \geq 2$

$2y^2 - 8y = x$

$y^2 - 4y = \frac{1}{2}x$

$y^2 - 4y + 2^2 = \frac{1}{2}x + 4$

$(y-2)^2 = \frac{1}{2}x + 4$

$y-2 = \pm\sqrt{\frac{1}{2}x+4}$

$y \geq 2 \Rightarrow y-2 \geq 0 \Rightarrow$

$\boxed{y = 2+\sqrt{\frac{1}{2}x+4} = f^{-1}(x)}$

202

(34) $y = \sqrt[3]{1-x^3} = g(x)$

(a) Find $g^{-1}(x)$:

$\sqrt[3]{1-y^3} = x$

$g = g^{-1}$, is its own inverse!?

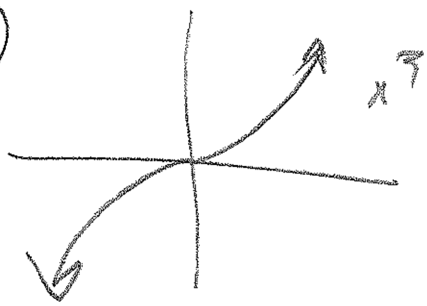
$1-y^3 = x^3$

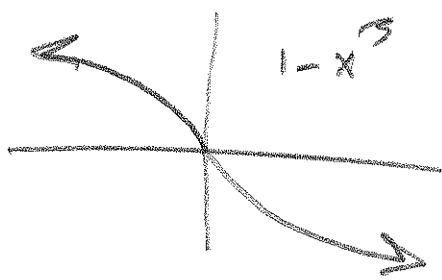
$-y^3 = x^3 - 1$

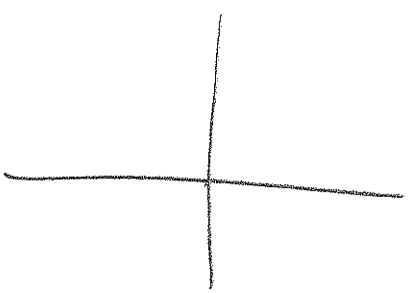
$y^3 = 1 - x^3$

$y = \boxed{\sqrt[3]{1-x^3} = g^{-1}(x)}$

(b)







202 §6.1

#s 35–42 are the meat of the new stuff

#s 35-38

(a) Show f is 1-to-1

(b) Use T7 to find $(f^{-1})'(a)$

(c) Calculate (Determine?) $f^{-1}(x)$ and give $\mathcal{D}$ & $\mathcal{R}$ of f^{-1}.

(d) Find $(f^{-1})'(a)$ using (c). Compare to (b).

(e) Sketch f & f^{-1} on same axes

35 $f(x) = x^3$, $a = 8$ $\qquad (b, a) = (b, 8) = (2, 8) =$

(a) $f(x_1) = f(x_2)$

$\Longrightarrow x_1^3 = x_2^3$

$\Longrightarrow \sqrt[3]{x_1^3} = \sqrt[3]{x_2^3}$

$\Longrightarrow x_1 = x_2$ ✓

$\blacksquare$

(b) $f'(x) = 3x^2$

$a = 8 = f(b) = b^3$

$\left(f^{-1}(a) = f^{-1}(8) = f^{-1}(b^3) = b \right)$

Want $f^{-1}(8)$, where $f(x) = x^3$

Method $x^3 = 8$, solve for x

$\sqrt[3]{x^3} = \sqrt[3]{8} = 2 = f^{-1}(a)$

$$\frac{1}{f'(f^{-1}(a))} = \frac{1}{f'(2)} = \frac{1}{3(2)^2} = \frac{1}{12}$$

$$\boxed{\frac{1}{12} = (f^{-1})'(8)}$$

202 §6.1 #s 35–42

35 cnt'd

(c) $y = x^3$

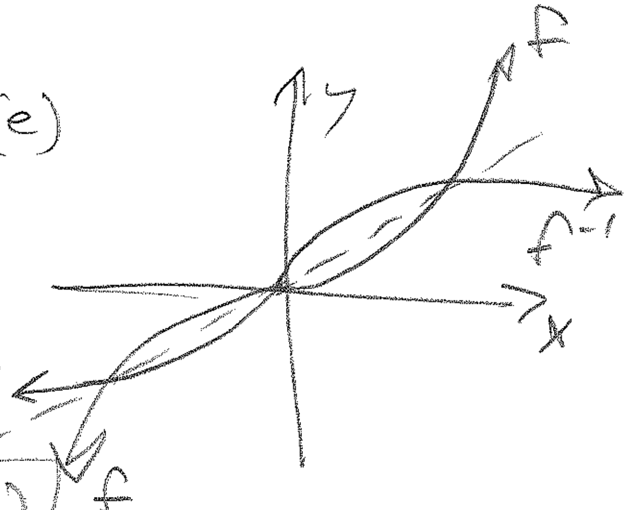
$y^3 = x$

$\sqrt[3]{y^3} = \sqrt[3]{x}$

$y = \boxed{\sqrt[3]{x} = f^{-1}(x)}$ f^{-1}

$$\boxed{\begin{array}{l} \mathcal{D}(f) = \mathcal{R}(f^{-1}) = \mathbb{R} = (-\infty, \infty) \\ \mathcal{R}(f) = \mathcal{D}(f^{-1}) = \mathbb{R} \end{array}}$$

(e)



(d) $(f^{-1})'(8) = ?$

$(f^{-1})(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$(f^{-1})'(x) = \frac{1}{3} x^{-\frac{2}{3}}$

$(f^{-1})'(8) = \frac{1}{3}(8)^{-\frac{2}{3}}$

$= \frac{1}{3}\left(\frac{1}{8}\right)^{\frac{2}{3}} = \frac{1}{3}\left(\left(\frac{1}{2^3}\right)^{\frac{1}{3}}\right)^2$

$= \frac{1}{3}\left(\frac{1}{2}\right)^2 = \frac{1}{3}\left(\frac{1}{4}\right) = \boxed{\frac{1}{12} = (f^{-1})'(8)}$

202 §6.1 #s 36-42

36) $f(x) = \sqrt{x-2}\,,\ a = 2$

(a) f is 1-to-1

if $f(x_1) = f(x_2)$

$\longrightarrow \sqrt{x_1-2} = \sqrt{x_2-2}$

$(\sqrt{x_1-2})^2 = (\sqrt{x_2-2})^2$

$x_1 - 2 = x_2 - 2$

$x_1 = x_2$ ■

(b) $a = 2 = \sqrt{x-2} \longrightarrow$

$4 = x-2$

$x-2 = 4$

$x = \underline{6 = f^{-1}(a) = b}$

$f(x) = (x-2)^{\frac{1}{2}} \longrightarrow$

$f'(x) = \frac{1}{2}(x-2)^{-\frac{1}{2}} \longrightarrow$

$f'(b) = f'(f^{-1}(a)) =$

$\frac{1}{2}(6-2)^{-\frac{1}{2}} = \frac{1}{2}(4)^{-\frac{1}{2}}$

$= \frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{2}} = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$

$f'(f^{-1}(a)) = \frac{1}{4}$, so

$(f^{-1})'(a) = \dfrac{1}{f'(f^{-1}(a))} = \dfrac{1}{\frac{1}{4}}$

$= \boxed{4 = (f^{-1})'(a)}$

(c) $y = \sqrt{x-2}$

$\sqrt{y-2} = x$

$y - 2 = x^2$

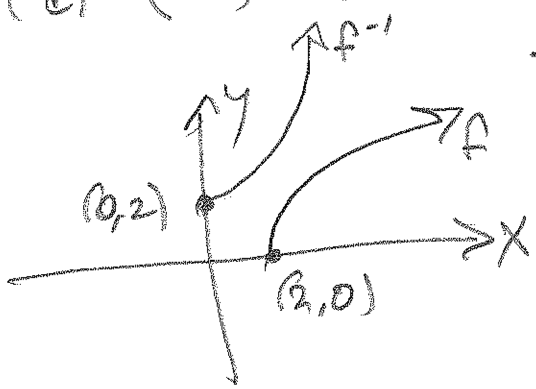
$y = \boxed{x^2 + 2 = f^{-1}(x)}$

$\boxed{\begin{array}{l} \mathcal{D}(f) = [2, \infty) = \mathcal{R}(f^{-1}) \\ \mathcal{R}(f) = [0, \infty) = \mathcal{D}(f^{-1}) \end{array}}$

(d) $(f^{-1})'(x) = 2x$

$(f^{-1})'(a) = (f^{-1})'(2) = 2(2) = \boxed{4 = (f^{-1})'(a)}$

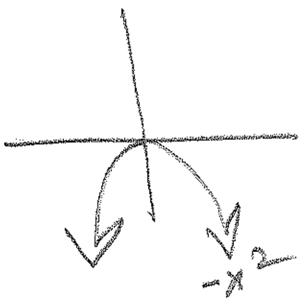
(e)

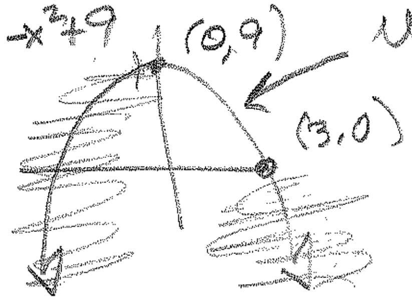


202 §6.1 #s 37–42

(37) $f(x) = -x^2 + 9$, $a = 8$

$\mathcal{D} = [0, 3]$





We want this part

$\mathcal{D} = [0,3] = \mathcal{R}(f^{-1})$

$\mathcal{R}(f) = [0,9] = \mathcal{D}(f^{-1})$

(a) f is 1-to-1:

If $f(x_1) = f(x_2)$ Then

$-x_1^2 + 9 = -x_2^2 + 9$

$-x_1^2 = -x_2^2$

$x_1^2 = x_2^2$

$x_1 = \pm x_2$

But $x_1, x_2 \in \mathcal{D}(f)$ means

$0 \leq x_1, x_2 \leq 3 \Rightarrow x_1 = -x_2$

doesn't happen! (except $x_1 = x_2 = 0$)

So 1-to-1 ■

(b) $f(x) = -x^2 + 9$

$f'(x) = -2x$

$a = 8 \Rightarrow -x^2 + 9 = 8$

$-x^2 = -1$

$x^2 = 1$

$x = \pm 1$ ($x \neq -1$, by $\mathcal{D}$)

$x = 1 = b = f^{-1}(a)$, so

$$\frac{1}{f'(f^{-1}(a))} = \frac{1}{f'(1)}$$

$$= \frac{1}{-2(1)} = -\frac{1}{2}$$

$$\boxed{(f^{-1})'(8) = -\frac{1}{2}}$$

202 § 6.1 #s 37-42

37 (c) $y = -x^2+9$

$$-y^2+9 = x$$
$$-y^2 = x-9$$
$$y^2 = -x+9$$
$$y = \pm\sqrt{-x+9}$$

$\Rightarrow y = \boxed{+\sqrt{-x+9} = f^{-1}(x)}$

We take $+\sqrt{-x+9}$, by $\mathcal{D}$ & R considerations, in particular, $R(f^{-1}) = \mathcal{D}(f) = [0,3]$, so $y = -\sqrt{-x+9}$ makes no sense!

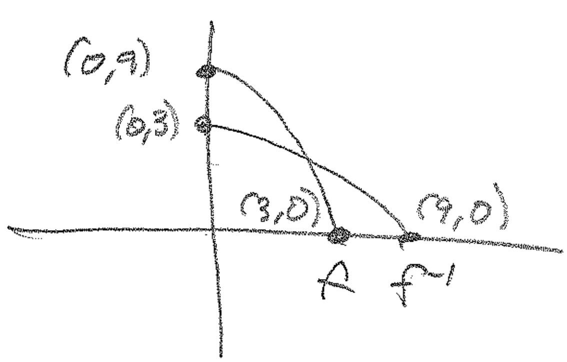
$$\boxed{\begin{array}{l} \mathcal{D}(f) = R(f^{-1}) = [0,3] \ \& \\ R(f) = \mathcal{D}(f^{-1}) = [0,9] \end{array}}$$

(d) $(f^{-1})(x) = \sqrt{-x+9} = (-x+9)^{\frac{1}{2}}$

$$\Rightarrow (f^{-1})'(x) = \tfrac{1}{2}(-x+9)^{-\frac{1}{2}}(-1)$$
$$= -\frac{1}{2\sqrt{-x+9}} \longrightarrow$$

$$(f^{-1})'(a) = (f^{-1})'(8) = \frac{-1}{2\sqrt{-8+9}} = \frac{-1}{2(\sqrt{1})} = -\frac{1}{2}$$

(e)



202 §6.1 #5 38–42

(38) $f(x) = \frac{1}{x-1}$, $x > 1$, $a = 2$

Find $f^{-1}(a)$:

$$\frac{1}{x-1} = 2$$

$$1 = 2(x-1) = 2x - 2$$

$$2x - 2 = 1$$

$$2x = 3$$

$$x = \boxed{\frac{3}{2} = f^{-1}(a)}$$

(a) f is 1-to-1

$$f(x_1) = f(x_2)$$

$$\frac{1}{x_1 - 1} = \frac{1}{x_2 - 1}$$

$$x_2 - 1 = x_1 - 1$$

$$x_2 = x_1 \quad \blacksquare$$

(b) $f(x) = \frac{1}{x-1} = (x-1)^{-1}$

$$f'(x) = -1(x-1)^{-2} = -\frac{1}{(x-1)^2}$$

$$(f^{-1})'(a) = (f^{-1})'(2)$$

$$= \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(\frac{3}{2})}$$

$$= \frac{1}{-\frac{1}{(\frac{3}{2}-1)^2}} = -(\tfrac{3}{2}-1)^2$$

$$= -(\tfrac{1}{2})^2 = \boxed{-\frac{1}{4} = (f^{-1})'(2)}$$

(c) $\frac{1}{y-1} = x$

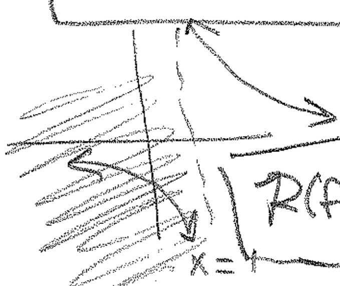
$$1 = x(y-1) = xy - x$$

$$-xy = -x - 1$$

$$xy = x + 1$$

$$y = \boxed{\frac{x+1}{x} = f^{-1}(x)}$$

$$\boxed{\mathcal{D}(f) = (1, \infty) = \mathcal{R}(f^{-1})}$$



$$\boxed{\mathcal{R}(f) = (0, \infty) = \mathcal{D}(f^{-1})}$$

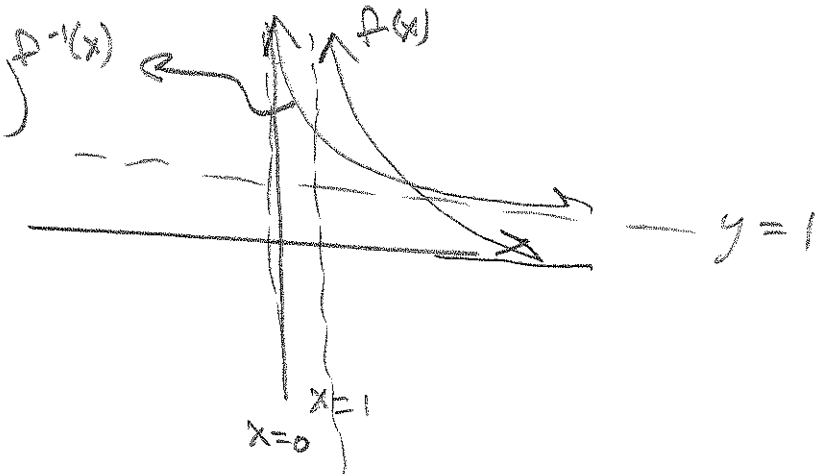
202 §6.1 #s 38-42

(38) $(f^{-1})(x) = \dfrac{x+1}{x}$, $x > 0$

$$(f^{-1})'(x) = \frac{1(x) - (x+1)(1)}{x^2} = \frac{x - x - 1}{x^2} = -\frac{1}{x^2}$$

$$\boxed{(f^{-1})'(2) = \frac{-1}{2^2} = -\frac{1}{4}}$$

(e)



$\dfrac{x+1}{x}$: V.A.: $x=0$
H.A.: $y=1$

$\dfrac{1}{x-1}$: V.A.: $x=1$
H.A.: $y=0$

202 §6.1 #s 39–42

39–42 Find $(f^{-1})'(a) =$

39 $f(x) = 2x^3+3x^2+7x+4$, $a=4$

Since $f(0)=4$, we have $(f^{-1})(a)=0$ &

$(0,4)$ is (b,a) on graph of $f(x)$

and $(4,0)$ is (a,b) on graph of $f^{-1}(x)$

a $\qquad f^{-1}(a)$

$f'(x) = 6x^2+6x+7 \longrightarrow$

$f'(f^{-1}(a)) = 6(0)^2+6(0)+7 = 7$

So

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))} = \boxed{\frac{1}{7}}$$

202 §6.1 #s 40–42

(40) $f(x) = x^3 + 3\sin x + 2\cos x$, $a = 2$

Again, they hand us $x = 0$ as $f^{-1}(a)$, since

$f(0) = 0^3 + 3\sin(0) + 2\cos(0) = 2$

So, $(0,2) = (b,a)$ on graph of f

and $(2,0) = (a,b) = (a, f^{-1}(a))$ is on f^{-1}.

$f'(x) = 3x^2 + 3\cos x - 2\sin x \longrightarrow$

$f'(f^{-1}(a)) = f'(0) = 3(0)^2 + 3\cos(0) - 2\sin(0)$

$= 3$. This gives

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \boxed{\frac{1}{3}}$$

(41) $f(x) = x^2 + \tan\left(\frac{\pi x}{2}\right) + 3$, $-1 < x < 1$, $a = 3$

AGAIN $b = f^{-1}(a) = 0$, since

$f(0) = 0^2 + \tan(0) + 3 = 3 = a$.

$f'(x) = 2x + \frac{\pi}{2}\sec^2\left(\frac{\pi}{2}x\right) \longrightarrow$

$f'(0) = 2 + \frac{\pi}{2}\sec^2(0) = 2 + \frac{\pi}{2}$

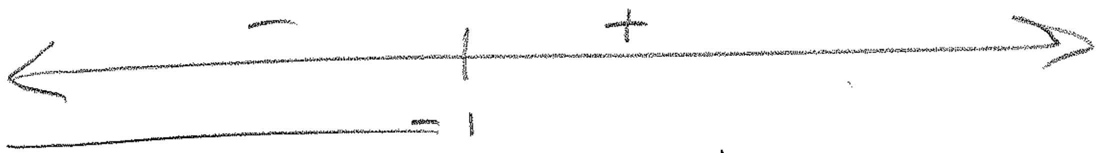
$$\therefore (f^{-1})'(3) = \frac{1}{2+\frac{\pi}{2}} = \frac{1}{\frac{4+\pi}{2}} = \boxed{\frac{2}{\pi+4} = (f^{-1})'(a)}$$

202 §6.1 #42

(42) $f(x)=\sqrt{x^3+x^2+x+1}$, $a=2$

$x^3+x^2+x+1 = x^2(x+1)+(x+1)$

$= (x+1)(x^2+1) \overset{\text{WANT}}{\geq} 0 \Longrightarrow$



$\boxed{\mathcal{D}(f) = [-1,\infty)} = \mathcal{R}(f^{-1})$

Let's "guess" $f(-1)=2$ Nope

Wan $(x+1)(x^2+1) = 4$, so $f(x)=2$

Oh. Duh. $x=1$ $\sqrt{1^3+1^2+1+1} = \sqrt{4} = 2$ ✓

So $(1,2) = (b,a)$ on f

$\Longrightarrow (2,1) = (a,b)$ on f^{-1} $b=4=(f^{-1})(a)$ in T7

$f'(x) = \frac{1}{2}(x^3+x^2+x+1)^{-\frac{1}{2}}(3x^2+2x+1)$

$f'(f^{-1}(a)) = f'(1) = \frac{1}{2}(1^3+1^2+1+1)^{-\frac{1}{2}}(3(1)^2+2(1)+1)$

$= \dfrac{3+2+1}{2\sqrt{1+1+1+1}} = \dfrac{6}{2\sqrt{4}} = \dfrac{6}{2\cdot 2} = \boxed{\dfrac{3}{2} = (f^{-1})'(2)}$