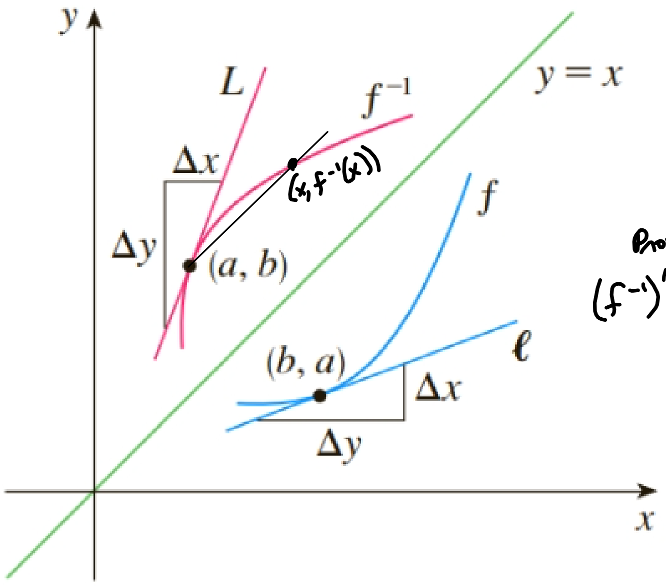
7 Theorem If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$



Let $y = f^{-1}(x) \Longrightarrow f(y) = x$

From graph:

$(a,b) = (a, f^{-1}(a))$

$(b,a) = (b, f(b))$

Prove the formula.

$$(f^{-1})'(a) = \lim_{x \to a} \frac{f^{-1}(x) - f^{-1}(a)}{x - a}$$

$$= \lim_{x \to a} \frac{y - b}{f(y) - f(b)}$$

f^{-1} is cont's, so $x \to a$ means $f^{-1}(x) \to f^{-1}(a)$

i.e., $y \to b$

$$= \lim_{y \to b} \frac{1}{\frac{f(y) - f(b)}{y - b}}$$

$$= \frac{1}{f'(b)} = \frac{1}{f'(f^{-1}(a))}$$