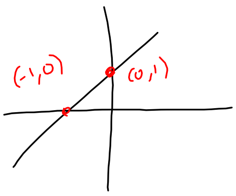
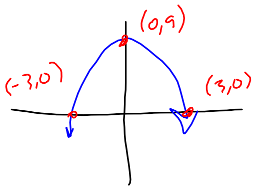
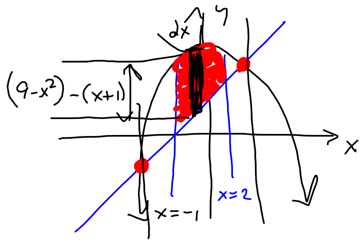
§5.1 Area Between 2 Curves

#s 5-12 sketch region bdd by given curves. Draw approximating rectangle. Find area.

(5) $y = x+1$, $y = 9-x^2$, $x=-1$, $x=2$







a b ≈ 2.37

$$\int_{-1}^{2}\left((9-x^2)-(x+1)\right)dx = \frac{39}{2}$$

$9-x^2 = x+1$

$x^2-8+x = 0$

$x^2+x-8=0$

$a=1,\ b=1,\ c=-8$

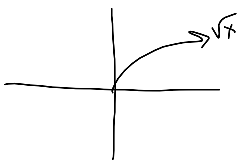
$b^2-4ac = 1^2-4(1)(-8)$

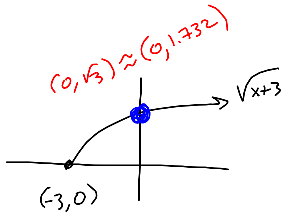
$= 1+32 = 33$

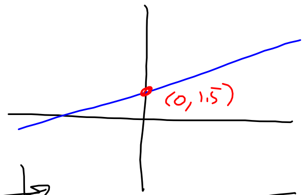
-1

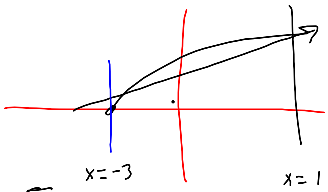
$x = \dfrac{-1\pm\sqrt{33}}{2(1)}$ → $\dfrac{-1+\sqrt{33}}{2} \approx 2.37$, $\dfrac{-1-\sqrt{33}}{2}$

9 $y=\sqrt{x+3}\ ,\ y=\frac{(x+3)}{2}=\frac{1}{2}x+\frac{3}{2}$









$\sqrt{x+3}=\frac{x+3}{2}$

$2\sqrt{x+3}=x+3$

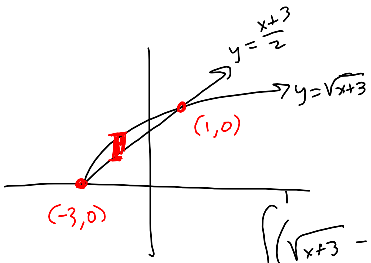
$4(x+3)=x^2+6x+9$

$4x+12=x^2+6x+9$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3,\ 1$



$$\int_{-3}^{1}\left(\sqrt{x+3}-\left(\frac{x+3}{2}\right)\right)dx$$

$$=\int_{-3}^{1}\sqrt{x+3}\,dx-\frac{1}{2}\int_{-3}^{1}(x+3)\,dx$$

$u=x+3$

$du=dx$

$\int u^{\frac{1}{2}}du$

$=\frac{2}{3}u^{\frac{3}{2}}+C$

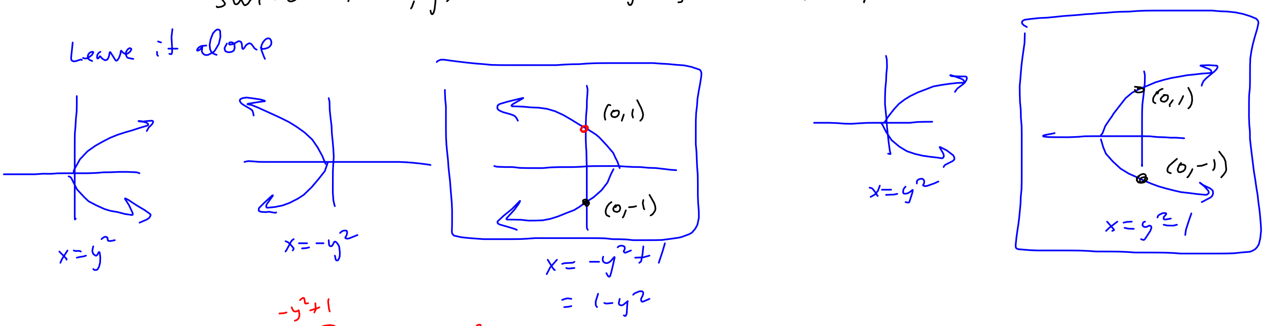
$$=\left[\frac{2}{3}(x+3)^{\frac{3}{2}}\right]_{-3}^{1}-\frac{1}{2}\left[\frac{1}{2}x^2+3x\right]_{-3}^{1}=\frac{4}{3}$$

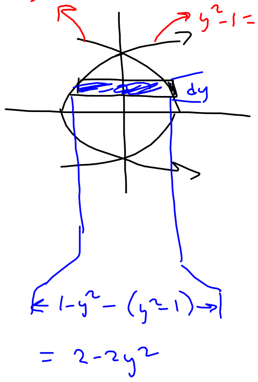
(11) $x = 1-y^2$, $x = y^2 - 1$

$= -y^2 + 1$

Switch x's & y's if $x = f(y)$ gives dyspepsia

Leave it alone





$-y^2+1 = x$

$-y^2 = x-1$

$y^2 = -x+1$

$y = \pm\sqrt{-x+1}$

$y = \sqrt{-x+1}$

$y^2 = x+1$

$y = \pm\sqrt{x+1}$

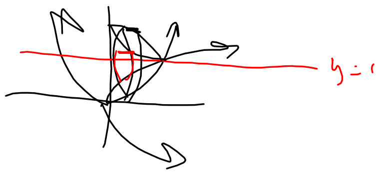
$$4\int_0^1 \sqrt{-x+1}\,dx$$

$$\int_{-1}^{1} (2-2y^2)\,dy = 2\int_0^1 (2-2y^2)\,dy = 4\int_0^1 (1-y^2)\,dy$$

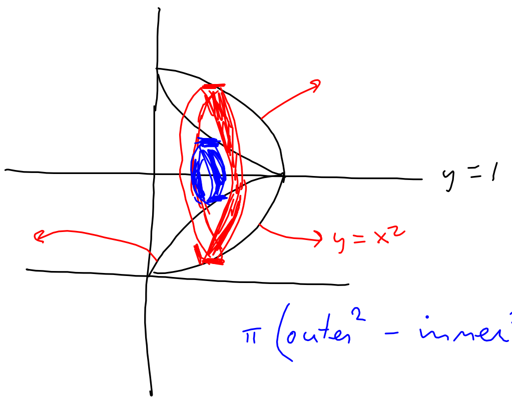
§5.2, 5.3 Volume of Solids of Revolution

#s 1-18 Find volume obtained by rotating the region bdd by the given curves about the specified line. Sketch region, solid & typical disk or washer.

11) $y = x^2$, $x = y^2$, about $y = 1$



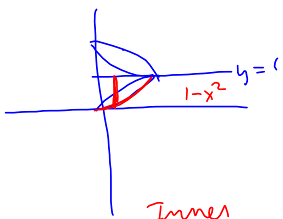
5.2 Disc (washer) Method



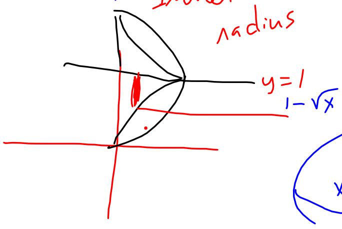
$\int_0^1 dx$

$\pi\left(\text{outer}^2 - \text{inner}^2\right)$

outer radius



Inner radius



$$\pi \int_a^b \left(\text{outer}^2 - \text{inner}^2\right) dx$$

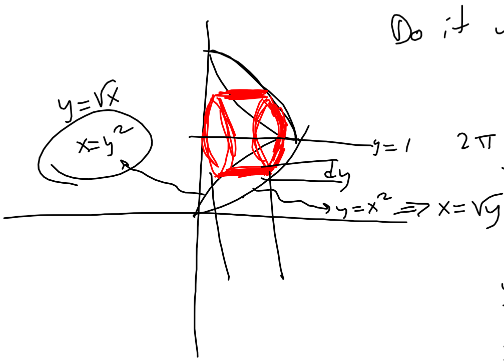
$$= \pi \int_0^1 \left((1-x^2)^2 - (1-\sqrt{x})^2\right) dx$$

$x = y^2$

$y = \pm\sqrt{x}$

$y = \sqrt{x}$

Do it w/ cylindrical shells



$2\pi \int \quad dy$

$y = x^2$

$x = y^2$

$\underline{2\pi r}\ \underline{h}\ \underline{\Delta y}$

$2\pi \int \underbrace{(1-y)}_{r} (\sqrt{y} - y^2)\, dy$

§5.4 work is force times distance
$F \cdot D$ = lb-ft or N-m. Acceleration of gravity is included in lbs, but not in kg. kg must be multiplied by 9.8 m/s² to get Newtons. English system, it's never about "slugs," the mass units in English system

(11) A spring has natural length 20cm. Compare the work W_1, done in stretching the spring from 20 to 30cm with the work W_2 done in stretching it from 30 to 40cm

Hooke's Law: $F = kx$ k is konstant (spring constant) is undetermined.

W_1 $x=0$, when length = 20
$x=10$, when " " 30

cm-g-s. We use m-kg-s

$$W_1 = \int_0^{10} kx\,dx = \left[\frac{k}{2}x^2\right]_0^{10} = \frac{100}{2}k = 50k = W_1$$

$x=10$ when $l=30$
$x=20$ " " " 40

$$W_2 = \int_{10}^{20} kx\,dx = \left[\frac{k}{2}x^2\right]_{10}^{20} = \frac{k}{2}(400) - \frac{k}{2}(100) = 200k - 50k = 150k$$

$W_2 = 150k$
$W_1 = 50k$, so $W_2 = 3W_1$

(16) Bucket weighs 4 lb, weightless rope.
80 ft well. 40 lbs H_2O in bucket.
Raising @ rate of 2ft/s
Water leaks @ .2 lb/s
Find work done in pulling bucket up.

$W = F \cdot D$

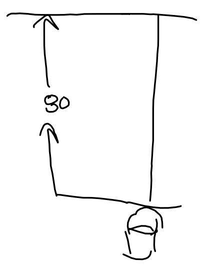
$F = 4 + 40 - \left(.2 \frac{lb}{\cancel{s}}\right)(t\ \cancel{s})$

$= 44 - .2t$

Move it a small distance

$F \cdot D = F \cdot \Delta x$

Total force over whole distance



$x = 80 \Leftrightarrow t = 40$

$$\int_{0=x}^{x=80} (44 - .2t)\, dx \nearrow \int_{t=0}^{t=40} (44 - .2t)(2dt)$$

$$\searrow \int_{x=0}^{x=80} \left(44 - .2\left(\tfrac{1}{2}x\right)\right) dx$$

$$= \int_0^8 (44 - .1x)\, dx$$

$x = \left(2 \frac{ft}{\cancel{s}}\right)(t\ \cancel{s}) = 2t$ ft

$dx = 2dt$

$x = 2t$

$t = \frac{1}{2}x$

$x = 80 = 2t \Rightarrow t = 40$

C5 in General: Setups are by far more important!
Pictures are important
Numerical Answers very minor.