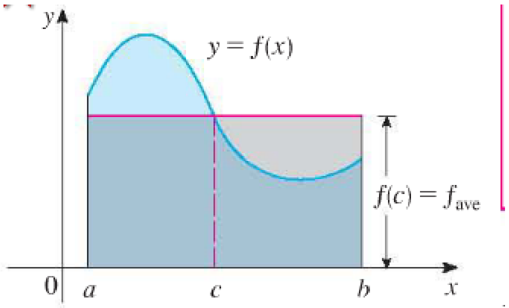
S5.5 Average Value of a function
(Mean Value Theorem for Integrals)

The Mean Value Theorem for Integrals If f is continuous on $[a, b]$, then there exists a number c in $[a, b]$ such that

$$f(c) = f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

that is, $$\int_a^b f(x)\,dx = f(c)(b-a) = f_{avg}\,(b-a)$$



Find average value of $f(x) = x^2 - 3x + 2$ on $[1,5]$

Followup: Find $c \ni f(c) = f_{avg}$.

$$f_{avg} = \frac{1}{5-1}\int_1^5 (x^2-3x+2)\,dx = \frac{1}{4}\left[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x\right]_1^5$$

$$= \frac{1}{4}\left[\frac{1}{3}(5^3) - \frac{3}{2}(5)^2 + 2(5) - \left(\frac{1}{3} - \frac{3}{2} + 2\right)\right]$$

$$= \frac{1}{4}\left[\frac{125}{3} - \frac{75}{2} + 10 - \frac{2-9+12}{6}\right]$$

$$= \frac{1}{4}\left[\frac{250 - 225 + 60 - (5)}{6}\right] = \frac{1}{4}\left[\frac{25+55}{6}\right] = \frac{1}{24}\left[80\right]$$

$$= \frac{40}{12} = \frac{20}{6} = \frac{10}{3} = f_{avg}.$$

Followup: Find $c \ni f(c) = f_{avg}$

$$x^2 - 3x + 2 \overset{SET}{=} f_{avg} = \frac{10}{3}$$

$$x^2 - 3x + 2 - \frac{10}{3} = 0$$

$$x^2 - 3x + \frac{6-10}{3} = 0$$

$$x^2 - 3x - \frac{4}{3} = 0$$

$$3x^2 - 9x - 4 = 0$$

$a = 3, b = -9, c = -4$

$3\,\overline{)129}$ 43

$$b^2 - 4ac = (-9)^2 - 4(3)(-4)$$
$$= 81 + 48$$
$$= 129$$

$$x = \frac{9 \pm \sqrt{129}}{2(3)} = \frac{9 \pm \sqrt{129}}{6}$$