$V = 2\pi r h \Delta r$

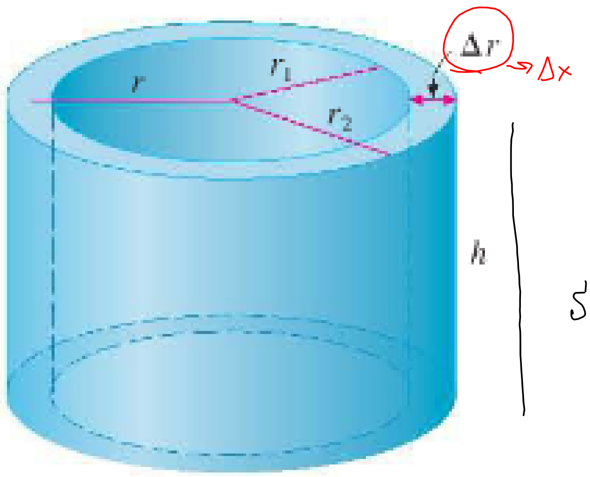


FIGURE 2

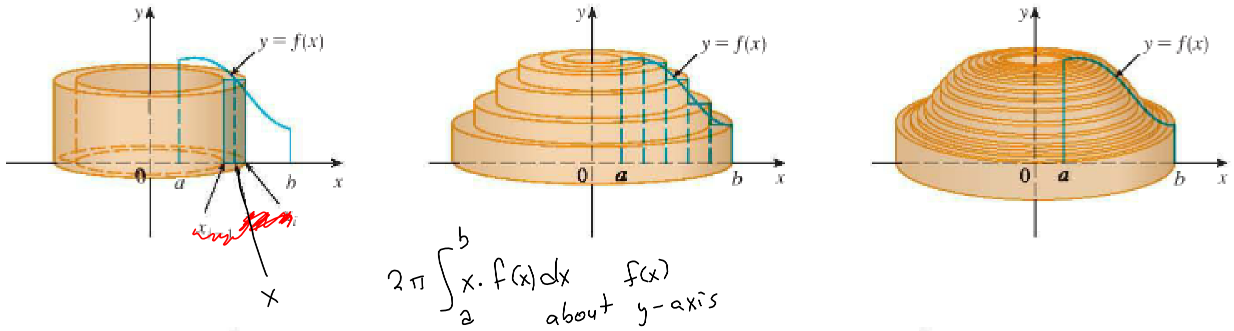
§5.3 Volumes of Solids of Revolution by cylindrical shells.

Volume of material used to make this section of clay pipe

$V = ?$

§5.2: Washer method would be $\pi r_2^2 h - \pi r_1^2 h$

But we're switching gears!



$$2\pi \int_a^b x \cdot f(x)\,dx \qquad f(x) \text{ about } y\text{-axis}$$

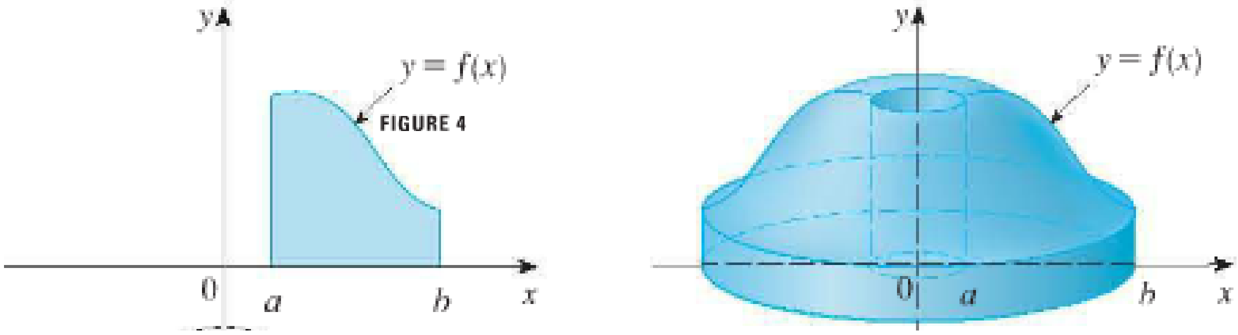
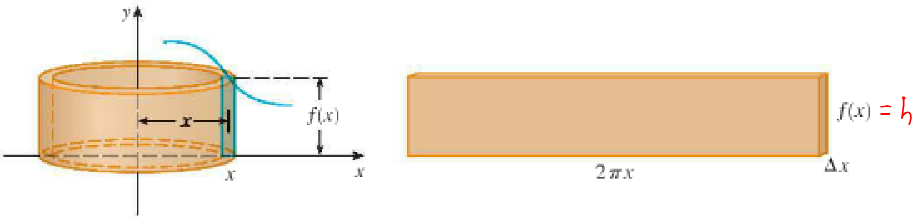


FIGURE 4

$$\int_a^b \underbrace{(2\pi x)}_{\text{circumference}} \; \underbrace{[f(x)]}_{\text{height}} \; \underbrace{dx}_{\text{thickness}}$$

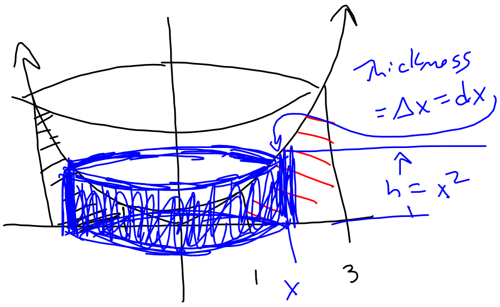


$\boxed{2}$ The volume of the solid in Figure 3, obtained by rotating about the y-axis the region under the curve $y = f(x)$ from a to b, is

$$V = \int_a^b 2\pi x f(x)\,dx \qquad \text{where } 0 \leqslant a < b$$

$f(x) = x^2$ about y-axis from $\boxed{x=1 \text{ to } x=3,\ y=0,\ y=x^2}$

Region bounded



$2\pi r h \Delta r$

$= 2\pi\, x \cdot x^2 \cdot dx$

$2\pi \int_1^3 x^3 dx = 2\pi \left[\frac{x^4}{4} \right]_1^3$

Sometimes Discs are better:

$y = f(x)$ about x-axis

$x = g(y)$ about y-axis

Sometimes Shells are Better.

$y = f(x)$ about y-axis

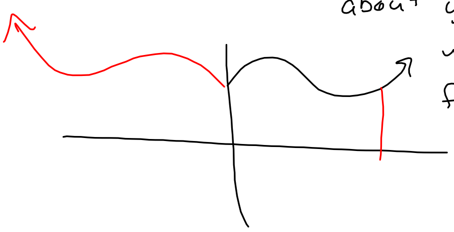
$x = g(y)$ about x-axis

Crossover: when $y = f(x)$ or $x = g(y)$ can be inverted.

$y = \sqrt[3]{x} \Longleftrightarrow x = y^3$

Shell method only:

about y-axis when $f(x)$ isn't 1-to-1



Disc method only

about x-axis

$y = f(x)$