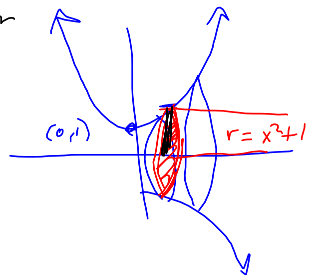
Rotate $y = x^2 + 1$ about the x-axis

Find volume between $x = 1$ and $x = 3$

volume of solid of revolution



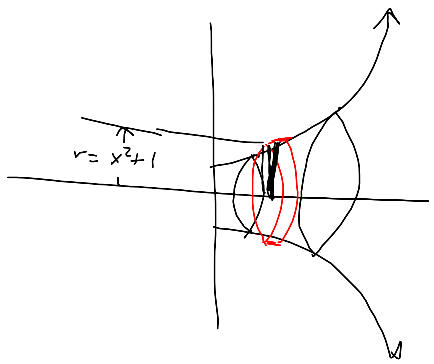
(Area of cross-section)(height)

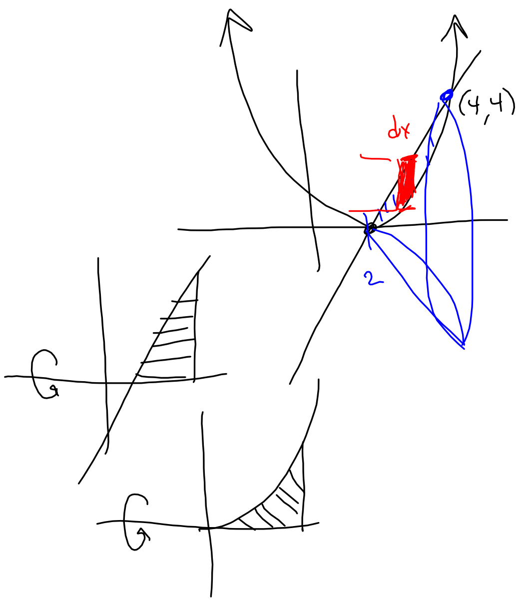
$(\pi r^2)(\Delta x)$

r comes from the function

$$\pi \int_1^3 (x^2+1)^2 \, dx = \pi \int_1^3 y^2 \, dx = \pi \int_1^3 f(x)^2 \, dx$$

$$= \pi \int_1^3 (x^4 + 2x^2 + 1) \, dx \text{, etc.}$$





$y = (x-2)^2 \qquad y = 2x-4 \qquad (0,-4) \quad (2,0)$

Volume when region between $y=(x-2)^2$ & $y=2x-4$ is revolved about x-axis

$(x-2)^2 = 2x-4$ $\qquad 2(4)-4=4$

$\Rightarrow x^2-4x+4 = 2x-4$

$\Rightarrow x^2-6x+8=0$

$\Rightarrow (x-2)(x-4)=0$

$\Rightarrow x=2,\ x=4 \longrightarrow \int_2^4$

V = Outer volume − Inner Volume = Volume of the washer

$$= \pi\int_2^4 (2x-4)^2\,dx - \pi\int_2^4 \left((x-2)^2\right)^2 dx$$

$$= \pi\int_2^4 \left((2x-4)^2 - (x-2)^4\right)dx = \text{etc.}$$

$\int u^2\,du = \frac{u^3}{3} + C$

$\pi\int_2^4 (2x-4)^2\,dx$ $\qquad u = 2x-4,\ du = 2dx$

$= \frac{1}{2}\pi\int_2^4 (2x-4)^2\cdot 2dx$

$= \frac{1}{2}\pi\left[\frac{(2x-4)^3}{3}\right]_2^4$

$x=2$, $u = 2(2)-4 = 0$

$x=4$, $u=2(4)-4=4$

$\frac{1}{2}\pi\int_0^4 u^2\,du = \frac{1}{2}\pi\left[\frac{u^3}{3}\right]_0^4$

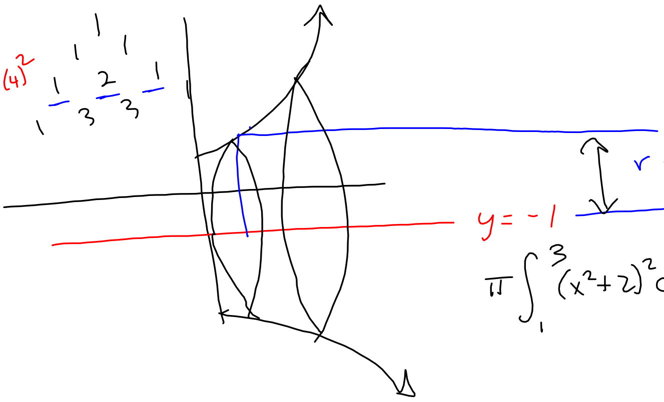
SAME!

$y = x^2 + 1$ about the line $y = -1$ $\quad x = 1$ to $x = 3$

$(x^2+4)^2$

1
1 1
1 2 1
1 3 3 1

$1x^4 + 2(x^2)(4) + 1(4)^2$



$y = -1$

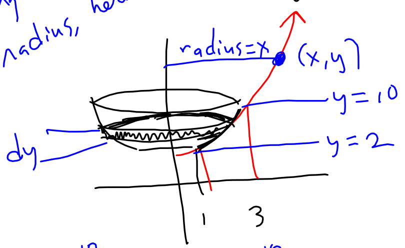
$r = x^2 + 1 - (-1) = x^2 + 2$

$$\pi \int_1^3 (x^2+2)^2\,dx = \pi \int_1^3 (x^4 + 8x^2 + 16)\,dx$$

Last wrinkle: Revolve about y-axis

The x is giving us the radius, here.

$y = x^2 + 1$, $x = 1$ to $x = 3$, about y-axis



This has discs on their side! stacked vertically!

$\pi \int \sim\sim dy$

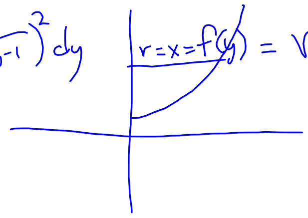
$y = x^2 + 1$

$y - 1 = x^2$

$x^2 = y - 1$

$x = \pm\sqrt{y-1}$

$V = \pi \int_2^{10} (f(y))^2 dy = \pi \int_2^{10} (\sqrt{y-1})^2 dy$



$= \pi \int_2^{10} (y-1)\, dy = etc$