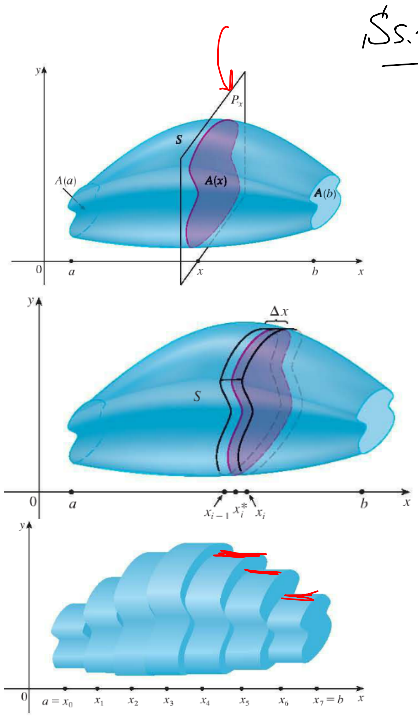


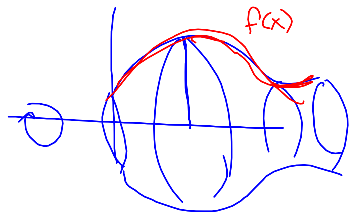
§5.2 Volumes

Volume of a loaf of bread is sum of volume of its slices!

Volume of a slice is its area times its thickness

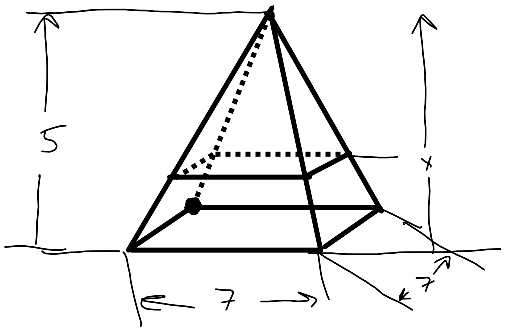
Our focus:

Disc Method & related washer method
for volume of solids of revolution



$$\text{Volume} = \sum_{k=1}^{n} A(x_k)\,\Delta x_k$$

Maybe not always the same thickness
(But we'll always do same thickness.)



Pyramid
Square Base
Height 5 m
width @ base is 7 m

Find cross-sectional area at some distance x from the base.

$$\text{Volume} \approx \sum_{k=1}^{n} A(x_k)\Delta x \xrightarrow{n\to\infty} \int_0^5 A(x)\,dx = \text{Volume}.$$

$A(x) = b^2$

$\sum_{k=1}^{n} A(x)\Delta x$

$= \sum_{k=1}^{n} b^2 \Delta x = \sum_{k=1}^{n} \left(-\frac{7}{5}x_k + 7\right)^2 \Delta x$

$\xrightarrow{n\to\infty} \int_0^5 \left(-\frac{7}{5}x + 7\right)^2 dx = \text{Volume}$

At $x = 0$, base $= b = 7 \rightsquigarrow (x_1, b_1) = (0, 7)$

At $x = 5$, base $= b = 0$ $\quad (x_2, b_2) = (5, 0)$

$m = \frac{0-7}{5-0} = -\frac{7}{5}$

view base as function of x!

$b = m(x - x_1) + b_1$

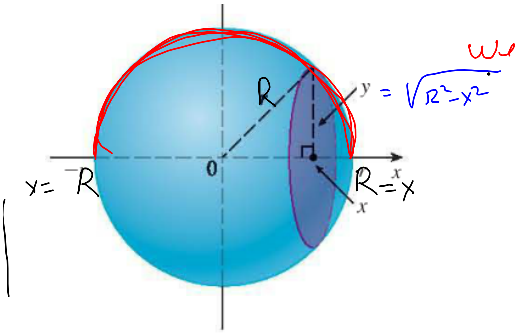
$= -\frac{7}{5}(x - 0) + 7$

$= -\frac{7}{5}x + 7$

$x = \frac{5}{2} \Rightarrow$

$b = -\frac{7}{5}\cdot\frac{5}{2} + 7$

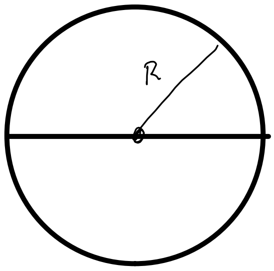
$= -\frac{7}{2} + 7 = \frac{7}{2}$



We show volume of the sphere of radius R

$$V = \frac{4}{3}\pi R^3$$

We revolve a semicircle about x-axis to obtain a sphere.



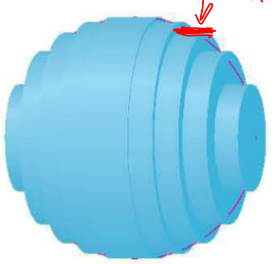
$x^2 + y^2 = R^2$

$y^2 = R^2 - x^2$

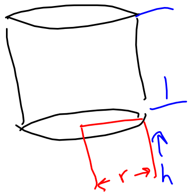
$y = \pm\sqrt{R^2 - x^2}$

Top $\frac{1}{2}$: $y = \sqrt{R^2 - x^2}$

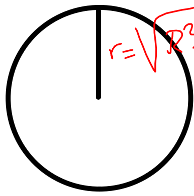
$h = \Delta x \to dx$



(b) Using 10 disks, $V \approx 4.2097$



$V = \pi r^2 h$



Area $= \pi r^2 = \pi\left(\sqrt{R^2 - x^2}\right)^2$

Add up the volume of all those slices that are Δx thick and have radius $r = \sqrt{R^2 - x^2}$

$$\sum \pi\left(\sqrt{R^2 - x^2}\right)^2 \Delta x = \pi \sum \sqrt{R^2 - x^2}\,\Delta x \xrightarrow{|\Delta x| \to 0} \pi\int_{-R}^{R} (R^2 - x^2)\,dx$$

$$\left(\sqrt{R^2 - x^2}\right)^2 = R^2 - x^2$$

$$\pi\int_{-R}^{R} (R^2 - x^2)\,dx = \pi\left[R^2 x - \frac{x^3}{3}\right]_{-R}^{R} = 2\pi\left[R^2 x - \frac{x^3}{3}\right]_{0}^{R}$$

$$= 2\pi \int_0^R (R^2 - x^2)\,dx$$

$$\int 7\,dx = 7x + C$$

$$= 2\pi\left[R^2(R) - \frac{R^3}{3}\right] = 2\pi\left[R^3 - \frac{R^3}{3}\right]$$

$$= 2\pi\left[\frac{2}{3}R^3\right] = \frac{4\pi}{3}R^3 \,!$$