S5.1 Area Between 2 Curves

Algebra, Trig, Analytic Geometry.

2 The area A of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the lines $x = a$, $x = b$, where f and g are continuous and $f(x) \geqslant g(x)$ for all x in $[a, b]$, is

$$A = \int_a^b [f(x) - g(x)]\, dx$$

3 The area between the curves $y = f(x)$ and $y = g(x)$ and between $x = a$ and $x = b$ is

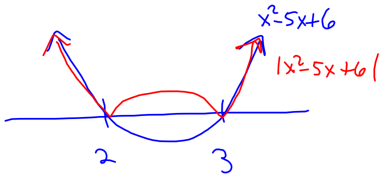
$$A = \int_a^b |f(x) - g(x)|\, dx$$

Find area bounded by $x=0, x=3$, $y=x^2-2x-3$ and $y=3x-9$

$$\int_0^3 \left| (x^2-2x-3)-(3x-9) \right| dx = \int_0^2 (x^2-5x+6)\,dx - \int_2^3 (x^2-5x+6)\,dx$$

$|x^2-2x-3-3x+9| = |x^2-5x+6|$

$|(x-2)(x-3)|$



How do we handle who's on top?

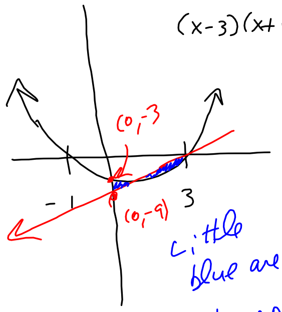
$x=2, x=3$ ← Solve $x^2-2x-3=3x-9$

or just write

$x^2-2x-3-(3x-9)$ & analyze its sign.

x^2-2x-3

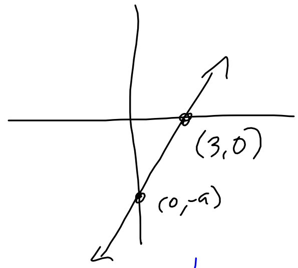
$(x-3)(x+1)$



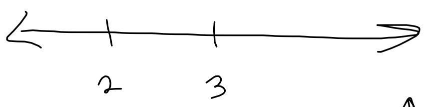
Little blue area?

We're not sure who's on top, where.

$3x-9$



$x^2-2x-3 = 3x-9$ when $x=2$, $x=3$



Test	x^2-2x-3	$3x-9$
$x=0$	-3	-9
$x=2.5$	-1.75	-1.5
$x=4$	5	3

on $(-\infty,2)$: x^2-2x-3 is on top

on $(2,3)$: $3x-9$ is on top

on $(3,\infty$: x^2-2x-3 " " "

That means

$|x^2-2x-3-(3x-9)|$

$= x^2-2x-3-(3x-9)$ on $(-\infty,2)\cup(3,\infty)$

& it $= (3x-9)-(x^2-2x-3)$ on $(2,3)$

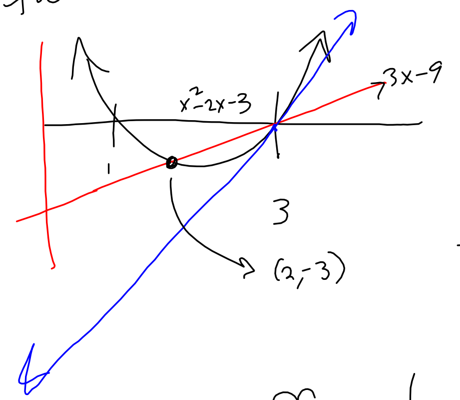
I usually just analyze f(x)−g(x) for its sign.

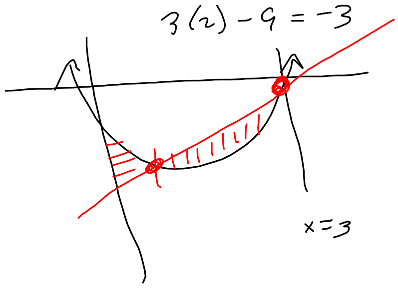
$$|x^2-2x-3-(3x-9)| = |x^2-5x+6|$$

Apply to sketch of the 2, together

$$= \begin{cases} x^2-5x+6 & \text{if } x \le 2 \text{ or } x \ge 3 \\ -(x^2-5x+6) & \text{if } 2 < x < 3 \end{cases}$$

$$\int_0^3 |f-g| = \int_0^2 (f-g) - \int_2^3 (f-g)$$





Way more efficient to just subtract the two funcs & graph |f − g|

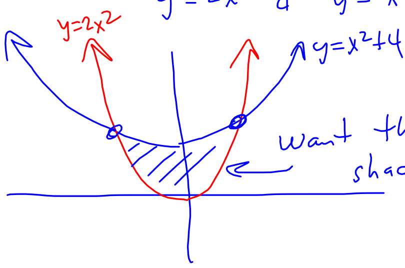
$$\int_0^2 ((x^2-2x-3)-(3x-9))\,dx$$

$$+\int_2^3 ((3x-9)-(x^2-2x-3))\,dx$$

Area Bounded by $x=2y^2$, $x=y^2+4$ #17

When in doubt, swap variables & work it this way:

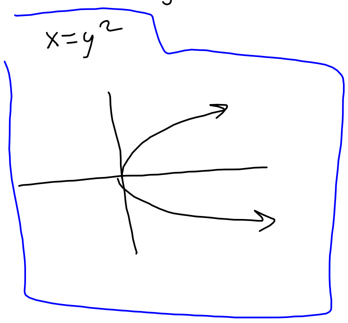
$y=2x^2$ & $y=x^2+4$



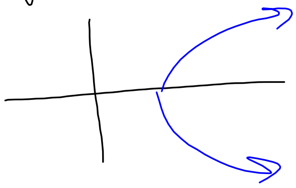
want the shaded area

$y=\pm\sqrt{x}$

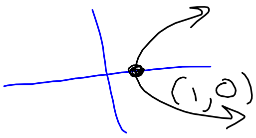
$x=y^2$



$x=y^2+4$



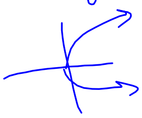
$x=y^2+1$



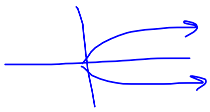
Short of a picture, you can always just go to town on

$\int|f-g|$

$x=y^2$



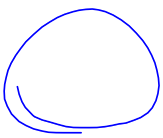
$x=2y^2$

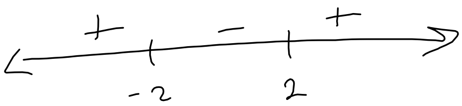


$x=2y^2$, $x=y^2+4$

$|2y^2-(y^2+4)| = |y^2-4|$

$= |y-2||y+2|$





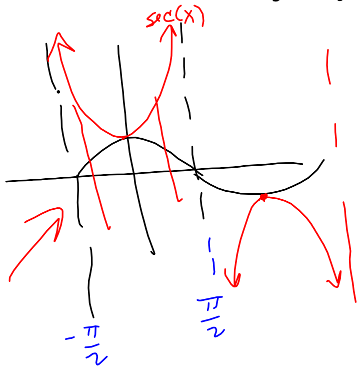
-2.1
0
2.1

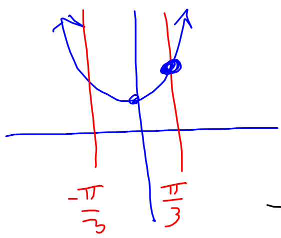
So this $|2y^2-(y^2+4)|$

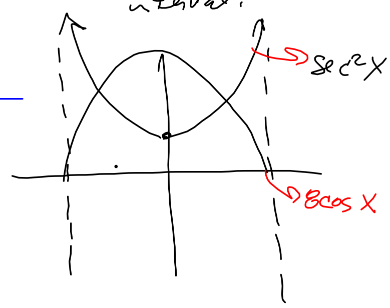
$= |y^2-4| =$ y^2-4 if $x\le -2$ or $x\ge 2$; $-(y^2-4)$ if $-2<x<2$

Area bounded by $y = \sec^2 x$, $y = 8\cos x$, $\underbrace{x = -\frac{\pi}{3},\ x = \frac{\pi}{3}}$ (#15)

$[-\frac{\pi}{3}, \frac{\pi}{3}]$ is symmetric interval.







Find intersection:

$$\sec^2 x = 8\cos x$$

$$\sec^2 x - 8\cos x = 0$$

$$\frac{1}{\cos^2 x} - 8\cos x =$$

$$\frac{1}{\cos^2 x} - \frac{8\cos x}{1} \cdot \frac{\cos^2 x}{\cos^2 x} = \frac{-8\cos^3 x + 1}{\cos^2 x} = 0$$

Same when $8\cos^3 x - 1 = 0$

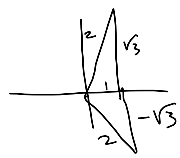
$$8\cos^3 x = 1$$

$$\sqrt[3]{8\cos^3 x} = \sqrt[3]{1}$$

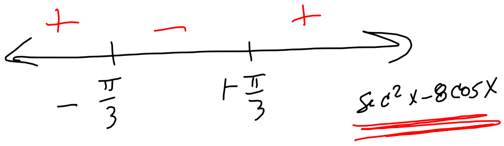
$$2\cos x = 1$$

$$\cos x = 1/2$$

How convenient! Handed us the intersection! $x = \pm\frac{\pi}{3}$



$-\frac{\pi}{3}$ is coterminal with $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$



So $|\sec^2 x - 8\cos x| = \boxed{8\cos x - \sec^2 x}$

& that's what we integrate,

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (8\cos x - \sec^2 x)\,dx$$

$$= 2\int_0^{\frac{\pi}{3}} (8\cos x - \sec^2 x)\,dx$$

by even function, symmetric interval.