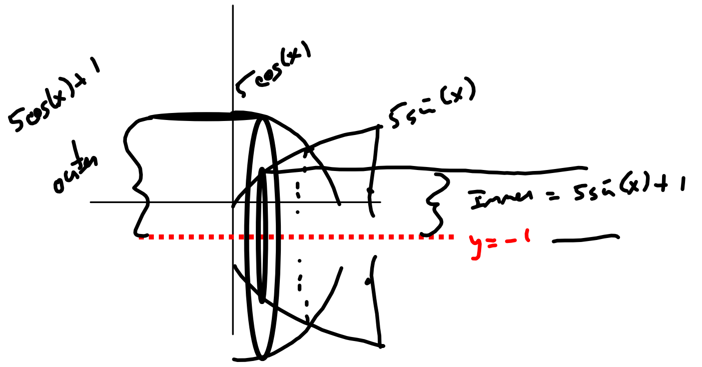
Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

#8

$$y = 5\sin(x),\ \ y = 5\cos(x),\ \ 0 \le x \le \frac{\pi}{4};\ \text{ about } y = -1$$

$V = \boxed{\pi\left(10\sqrt{2} + \frac{5}{2}\right)}$

Sketch the region.



$$V = \pi\int(\text{outer}^2 - \text{inner}^2)\,dx = \pi\int_0^{\frac{\pi}{4}}\left((5\cos(x)+1)^2 - (5\sin(x)+1)^2\right)dx$$

$$\pi\int_0^{\frac{\pi}{4}}\left(5\cos^2(x) - 5\sin^2(x)\right)dx$$

does ~~NOT~~ get it done.

Scratch:

$$(5\cos(x)+1)^2 = 25\cos^2(x) + 10\cos(x) + 1$$

$$-\quad (5\sin(x)+1)^2 = 25\sin^2(x) + 10\sin(x) + 1$$

$$= 25\underbrace{(\cos^2(x) - \sin^2(x))}_{\cos(2x)} + 10\cos(x) - 10\sin(x)$$

$$= 25\cos(2x) + 10\cos(x) - 10\sin(x)$$

$$\Rightarrow V = \pi\int_0^{\frac{\pi}{4}}\left(25\cos(2x) + 10\cos(x) - 10\sin(x)\right)dy$$

$$= 25\pi\cdot\frac{1}{2}\int_0^{\frac{\pi}{4}}\cos(2x)\cdot 2\,dx + 10\pi\Big[\sin(x)\Big]_0^{\frac{\pi}{4}} + 10\pi\Big[\cos(x)\Big]_0^{\frac{\pi}{4}}$$

$$= \frac{25\pi}{2}\Big[\sin(2x)\Big]_0^{\frac{\pi}{4}} + 10\pi\left[\sin\left(\frac{\pi}{4}\right) - 0\right] + 10\pi\left[\cos\left(\frac{\pi}{4}\right) - \cos(0)\right]$$

$$= \frac{25\pi}{2} + 10\pi\cdot\frac{\sqrt{2}}{2} + 10\pi\left[\frac{\sqrt{2}}{2} - 1\right]$$

$$= \frac{25\pi}{2} + \frac{20\sqrt{2}\pi}{2} - 10\pi = \boxed{\frac{5\pi}{2} + 10\sqrt{2}\pi = \text{Volume}}$$

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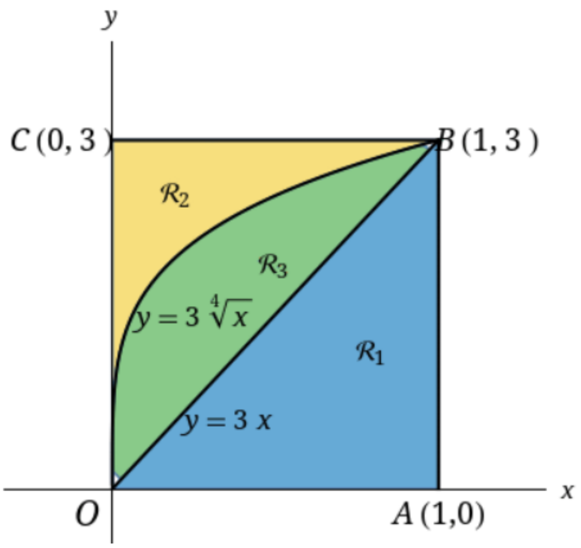
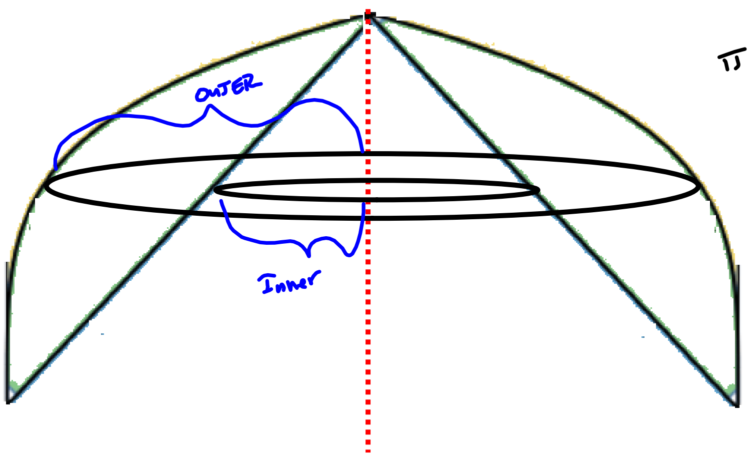
Three regions are defined in the figure.



Find the volume generated by rotating the given region about the specified line.

$\mathcal{R}_3$ about AB



$\pi \int_a^b (\text{OUTER}^2 - \text{INNER}^2)\, dy$

OUTER = $1 - x$, where

$x = ?$

$y = 3\sqrt[4]{x}$

$\rightarrow \sqrt[4]{x} = \frac{y}{3}$

$x = \left(\frac{y}{3}\right)^4$

Inner: $y = 3x \rightarrow$

$x = \frac{y}{3}$

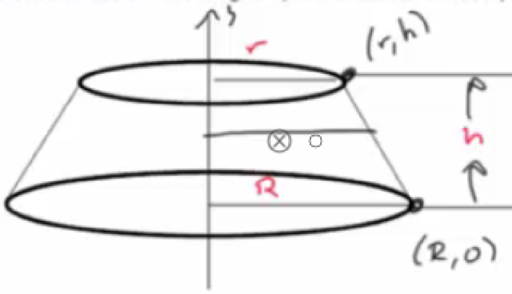
Volume $= \pi \int_0^3 \left(\left(1 - \left(\frac{y}{3}\right)^4\right)^2 - \left(1 - \frac{y}{3}\right)^2 \right) dy$ is the setup!

$\left(\left(\frac{y}{3}\right)^4 - 1\right)^2 = \frac{y^8}{9} - 2\left(\frac{y}{3}\right)^4(1) + 1 = \text{OUTER}^2$

14. Question Details SCalc8 5.2.048

Find the volume V of the described solid S.

A frustum of a right circular cone with height h, lower base radius R, and top radius r



$$m = \frac{h-0}{r-R} = \frac{h}{r-R}$$

$$y = \frac{h}{r-R}(x-R)$$

$$= m(x-x_1)+y_1$$

$$= \frac{h}{r-R}x - \frac{h}{r-R}R$$

$$\frac{h}{r-R}x = y + \frac{h}{r}$$

$$\pi\int_0^h \left(\frac{y(r-R)}{h} + R\right)^2 dy$$

$$= \pi\int_0^h \left(\left(\frac{y(r-R)}{h}\right)^2 + 2R\left(\frac{y(r-R)}{h}\right) + R^2\right) dy$$

$$= \pi\left[\frac{y^3}{3}\left(\frac{r-R}{h}\right)^2 + \frac{y^2}{2}\left(2R\left(\frac{r-R}{h}\right)\right) + R^2 y\right]_0^h$$

$$= \pi\left[\frac{\cancel{h^3}^{\,h}}{3}\left(\frac{R^2 - 2rR + r^2}{\cancel{h^2}_{\,1}}\right) + \frac{\cancel{h^2}^{\,h}}{2}\left(\frac{2Rr - 2R^2}{\cancel{h}_{\,1}}\right) + R^2 h\right]$$

$$= \pi\left[\frac{R^2}{3}h - \frac{2Rr}{3}h + \frac{r^2}{3}h + \frac{2Rr}{2}h - \frac{2R^2}{2}h + R^2 h\right]$$

$$= \pi h\left[\frac{R^2}{3} - \frac{2Rr}{3} + \frac{r^2}{3} + rR - R^2 + R^2\right]$$

$$= \pi h\left[\frac{R^2}{3} + \frac{Rr}{3} + \frac{r^2}{3}\right] = \boxed{\frac{\pi h}{3}\left[R^2 + Rr + r^2\right] = \text{VOLUME}}$$

$$-\frac{2Rr}{3}h + \frac{2Rr}{2}h$$

$$= \frac{-4Rrh + 6Rrh}{6}$$

$$= \frac{2Rrh}{6} = \frac{Rrh}{3}$$