Warm up with a couple typical puzzlers. The point is to not be discouraged and to try different things. Also, to be mindful of the domain of the integrand, before applying FTC II all willy-nilly

Antiderivative

$\sec^2\theta \longmapsto \tan\theta$

derivative antiderivative

33. $\displaystyle\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta$

34. $\displaystyle\int_0^{\pi/3} \frac{\sin\theta + \sin\theta\,\tan^2\theta}{\sec^2\theta}\,d\theta$

scratch:

$1-\cos^2\theta = \sin^2\theta$

$1-\sin^2\theta = \cos^2\theta$

$2-\sin^2\theta = \cos^2\theta + 1$

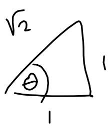
33 $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\cos^2\theta+1}{\cos^2\theta}\,d\theta$

$$\int \frac{2-\sin^2\theta}{\cos^2\theta}\,d\theta = \int\left[\frac{2}{\cos^2\theta} - \tan^2\theta\right]d\theta$$
$$= \int\left[2\sec^2\theta - \tan^2\theta\right]d\theta$$

$$\frac{\cos^2\theta+1}{\cos^2\theta} = \boxed{\frac{\cos^2\theta}{\cos^2\theta} + \frac{1}{\cos^2\theta}} = 1+\sec^2\theta$$ Aha!

$$\int_0^{\frac{\pi}{4}} \frac{\cos^2\theta+1}{\cos^2\theta}\,d\theta = \int_0^{\frac{\pi}{4}}(1+\sec^2\theta)\,d\theta = \Big[\theta + \tan\theta\Big]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4} + \tan\frac{\pi}{4} - \underbrace{(0+\tan 0)}_{0} = \frac{\pi}{4} + \frac{1}{\sqrt{2}} = \frac{\pi}{4} + \frac{\sqrt{2}}{2}$$



$$\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

33. $\displaystyle\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta$

34. $\displaystyle\int_0^{\pi/3} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\,d\theta$

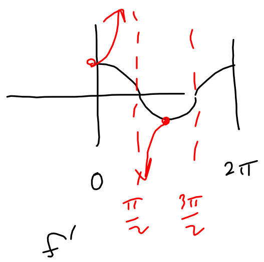
#34 $\dfrac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}$

$\dfrac{\sin\theta}{\sec^2\theta} + \dfrac{\sin\theta\tan^2\theta}{\sec^2\theta}$

$= \sin\theta\cos^2\theta + \sin\theta \cdot \dfrac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta$

$= \sin\theta\cos^2\theta + \sin^3\theta$

$= \dfrac{\sin\theta(1+\tan^2\theta)}{\sec^2\theta} = \dfrac{\sin\theta\sec^2\theta}{\sec^2\theta} = \sin\theta$



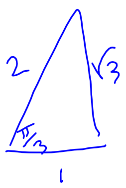
f	f'
$\cos\theta$	$-\sin\theta$
$\sin\theta$	$\cos\theta$

34 $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\,d\theta$

$$= \int_0^{\frac{\pi}{3}} \frac{\sin\theta(1+\tan^2\theta)}{\sec^2\theta}\,d\theta = \int_0^{\frac{\pi}{3}} \frac{\sin\theta(\sec^2\theta)}{\sec^2\theta}\,d\theta$$

$$= \int_0^{\frac{\pi}{3}} \sin\theta\,d\theta = \Big[-\cos\theta\Big]_0^{\frac{\pi}{3}} = -\cos\tfrac{\pi}{3} - (-\cos(0))$$

$$= -\tfrac{1}{2} + 1 = \boxed{\tfrac{1}{2}}$$



$\dfrac{d}{d\theta}\big[\cos\theta\big] = -\sin\theta$

$\dfrac{d}{d\theta}\big[-\cos\theta\big] = +\sin\theta$

Net Change Theorem The integral of a rate of change is the net change:

$$\int_a^b F'(x)\,dx = \underline{F(b) - F(a)}$$

	rate	Net change	
Kilo Watts	Power	Energy	Kilowatt-hours
$\frac{\text{miles}}{\text{hr}}$	Speed	Distance	miles.