§4.3 Fundamental Theorem of Calculus.

The derivative of the antiderivative is the integrand.

$$\int_1^3 x\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x = \int_a^b f(x)\,dx$$

$$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n}\left(\frac{2k}{n}+1\right)\frac{2}{n} = \frac{2}{n}\sum_{k=1}^{n}\left(\frac{2k}{n}+1\right) = \frac{2}{n}\left[\sum_{k=1}^{n}\frac{2k}{n} + \sum_{k=1}^{n} 1\right]$$

$f(x) = x$

$\Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$

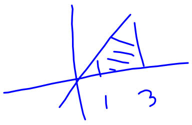
$x_k = a + k\Delta x = 1 + k\cdot\frac{2}{n} = \frac{2k}{n} + 1$

$$= \frac{2}{n}\left[\frac{2}{n}\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1\right]$$

$$= \frac{4}{n^2}\sum k + \frac{2}{n}\sum 1$$

$$= \frac{4}{n^2}\cdot\frac{n^2+\cdots}{2} + \frac{2}{n}\cdot n$$

$$\xrightarrow{n\to\infty} \frac{4n^2}{2n^2} + 2 = \frac{4}{2}+2 = 2+2 = 4$$



$f(x) = x$

$F(x) = \frac{x^2}{2}$ & $F(3) - F(1) = \frac{3^2}{2} - \frac{1^2}{2} = \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = 4$

Got $\int_1^3 x\,dx$ using an antiderivative.

In general, $F(x) = \frac{x^2}{2} + 7$ OR $\frac{x^2}{2} - \pi$ OR ...

FTC II

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Assume f is cnt^s on $[a, b]$

Some nonexamples in homework

$$\int_{-1}^{3} \frac{1}{x^2}\,dx = \left.\frac{x^{-1}}{-1}\right]_{-1}^{3} = \frac{3^{-1}}{-1} - \frac{1^{-1}}{-1} = -\frac{1}{3} + 1 = \frac{4}{3}$$

$\frac{2}{3}$

Not cnt^s @ $x=0$ NOT

FTC I   If f is cnts on $[a,b]$, then

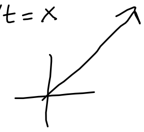
$\int_a^x f(t)\,dt$ is a function of x that's difbl,

and $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$

Integrand evaluated at x, $\int_0^0$

If $f(x) > 0$, then $\int_a^x f(t)\,dx$ is increasing.

$\int_0^x 1\,dt = x$



Chain Rule!

$$\frac{d}{dx}\int_1^{x^2} f(t)\,dt = f(x^2)\cdot 2x$$

Integrals with variable limit(s) of integration are functions of those variables

$$G(x) = \int_1^x f(t)\,dt$$

$$G(x) = \int_1^x t\,dt = \frac{t^2}{2}\Big]_1^x = \frac{x^2}{2} - \frac{1^2}{2}$$

$$G(x^2) = \int_1^{x^2} f(t)\,dt$$

we did $G(3)$, already

$$G(3) = \frac{3^2}{2} - \frac{1^2}{2}$$

$$G(4) = \frac{4^2}{2} - \frac{1^2}{2}$$

$$\boxed{\frac{d}{dx}\int_1^{\sin x} t^5 \cos^2(t)\,dt}$$

$$= (\sin x)^5 \cos^2(\sin x)\cdot \cos x$$

$$\int_1^{x^3} t\,dt = \frac{t^2}{2}\Big]_1^{x^3} = \frac{(x^3)^2}{2} - \frac{1}{2}$$

$$= \frac{x^6}{2} - \frac{1}{2}$$

So, $\frac{d}{dx}\int_1^{x^3} t\,dt = \frac{6x^5}{2} = 3x^5$

FTC I sez: $x^3 \cdot 3x^2 = 3x^5$

Chain Rule part

A little § 4.2

I left out this comparison in lecture, yesterday

ß f, g cnt$^{\underline{s}}$ & $f(x) \geq g(x)$ on $[a,b]$.

Then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

$$f(x) \geq 0 \Rightarrow \int_a^b f(x)\,dx \geq 0$$