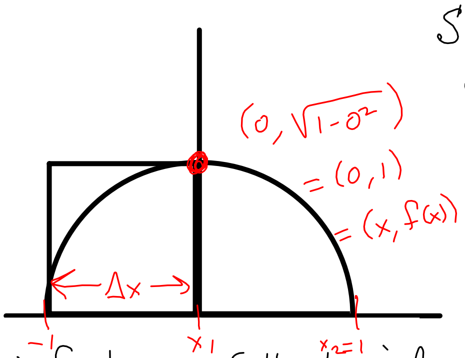
§4.1



Circle of radius $r = 1$.

Area $= A = \pi r^2 = \pi$

Eq'n is $x^2 + y^2 = 1$

$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$ → $y = +\sqrt{1-x^2}$ is top half.

We find area of the $\frac{1}{2}$-circle

$\frac{\pi}{2} \approx \frac{3.14}{2} = 1.57$

So, $y = \sqrt{1-x^2} = f(x)$

Approximate it with sum of the areas of rectangles.

Use 2 rectangles, Right Endpoints

Area = Sum of areas of rectangles

$= l \cdot w + l \cdot w$

$= 1 \cdot 1 + 0 \cdot 1$

$= 1$

$= f(x_1) \cdot \Delta x + f(x_2) \cdot \Delta x$

$= \sum_{k=1}^{2} f(x_k) \Delta x$

$= \sum_{k=1}^{2} \sqrt{1 - x_k^2}\, \Delta x$

$[-1, 1]$ = interval $= [a, b]$

$\Delta x = \frac{b-a}{2}$

$x_1 = a + \Delta x = -1 + 1 = 0$

$x_2 = a + 2\Delta x = -1 + 2 = 1$

In general

$\Delta x = \frac{b-a}{n}$

$x_k = a + k\Delta x$

$= \sqrt{1-x_1^2} \cdot 1 + \sqrt{1-x_2^2} \cdot \Delta x$

$= \sqrt{1-0^2} + \sqrt{1-1^2}$

$= \sqrt{1} + 0 = \sqrt{1} = 1$

Use 10 rectangles:

$n = 10$

$\Delta x = \frac{b-a}{n} = \frac{1-(-1)}{10} = \frac{2}{10} = \frac{1}{5} = \Delta x$

$x_k = k\Delta x = k \cdot \frac{1}{5} = \frac{1}{5}k = \frac{k}{5}$

Not $k\Delta x$

$a + k\Delta x$

$-1 + k\Delta x$

$= -1 + \frac{k}{5}$

$\therefore$ Area Approximation is

$$\sum_{k=1}^{10} f(x_k)\Delta x = \sum_{k=1}^{10} \sqrt{1 - x_k^2} \cdot \Delta x \quad \text{GOOD}$$

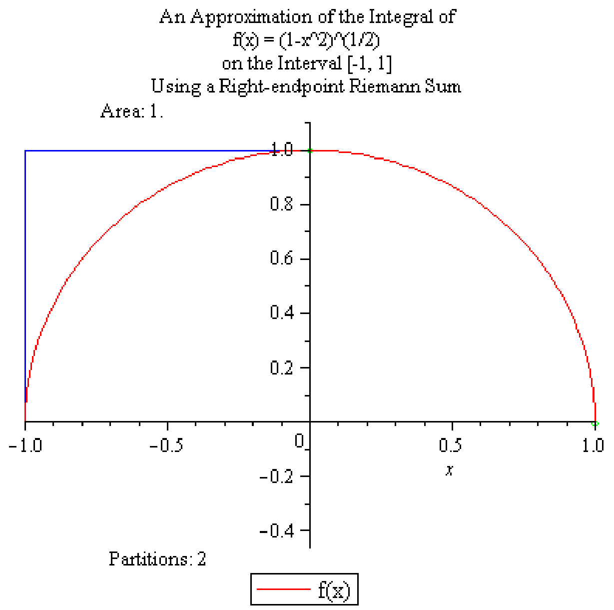
$$= \sum_{k=1}^{10} \sqrt{1 - \left(\frac{k}{5}\right)^2} \cdot \frac{k}{5} = \frac{1}{5} \sum \sqrt{1 - \left(\frac{k}{5}\right)^2} \cdot k$$

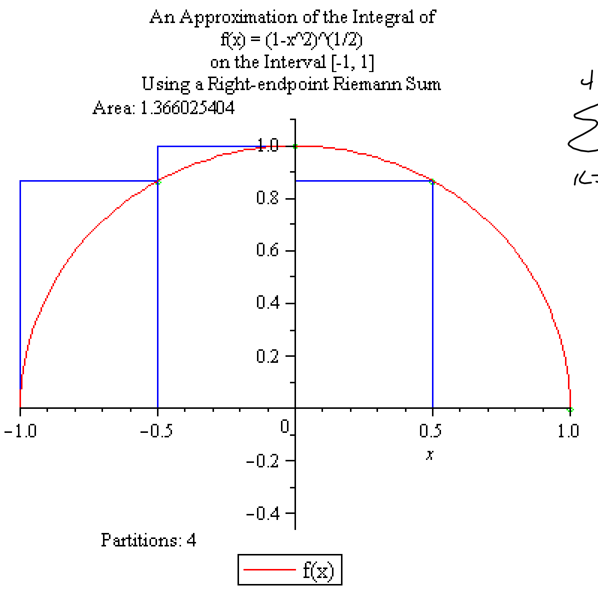
$$= \frac{1}{5} \sum_{k=1}^{10} k\sqrt{1 - \left(\frac{k}{5}\right)^2}$$

$$S_{10} = \frac{1}{5}\left[1\sqrt{1 - \left(\frac{1}{5}\right)^2} + 2\sqrt{1 - \left(\frac{2}{5}\right)^2} + 3\sqrt{1 - \left(\frac{3}{5}\right)^2} + \dots + 10\sqrt{1 - \left(\frac{10}{5}\right)^2}\right]$$

$$\sum_{k=1}^{n} f(x_k)\Delta x \qquad x_k = a + k \cdot \frac{b-a}{n}$$

$$\Delta x = \frac{b-a}{n}$$


An Approximation of the Integral of
f(x) = (1-x^2)^(1/2)
on the Interval [-1, 1]
Using a Right-endpoint Riemann Sum
Area: 1.
1.0
0.8
0.6
0.4
0.2
-1.0
-0.5
0
0.5
1.0
x
-0.2
-0.4
Partitions: 2
f(x)

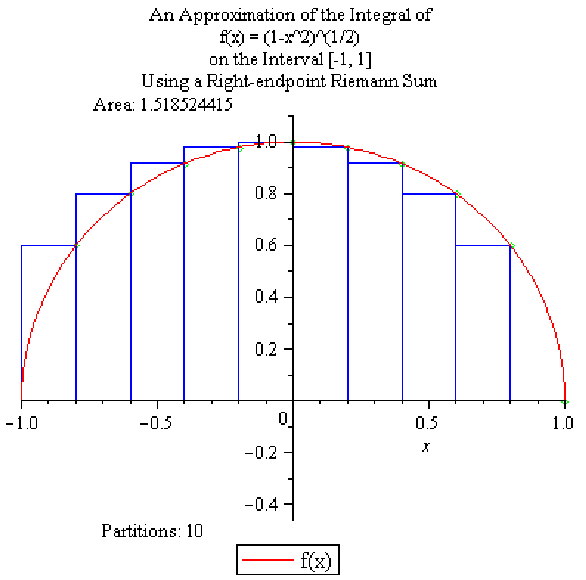


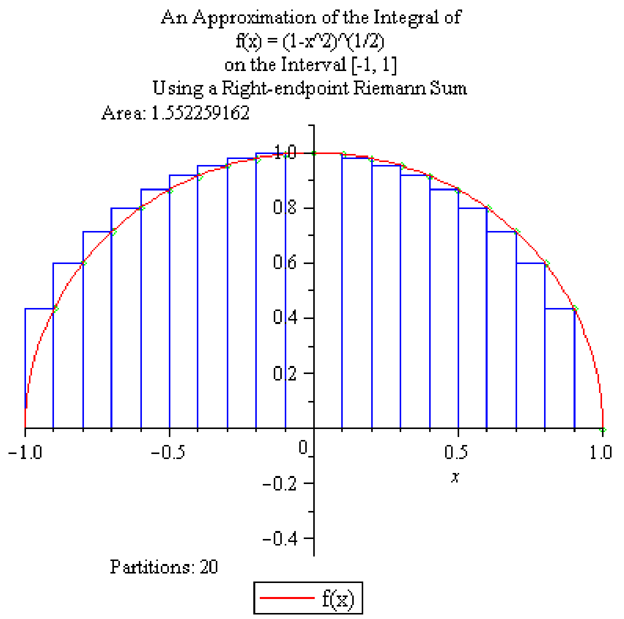
$$\sum_{k=1}^{4} k\sqrt{1-x_k^2} = \sum_{k=1}^{4} k\sqrt{1-\left(-1+\frac{k}{2}\right)^2}$$

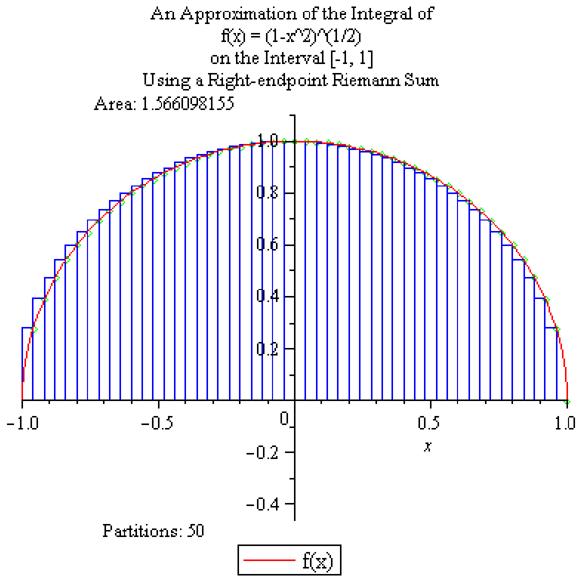
$$\Delta x = \frac{b-a}{n} = \frac{1-(-1)}{4} = \frac{2}{4} = \frac{1}{2}$$

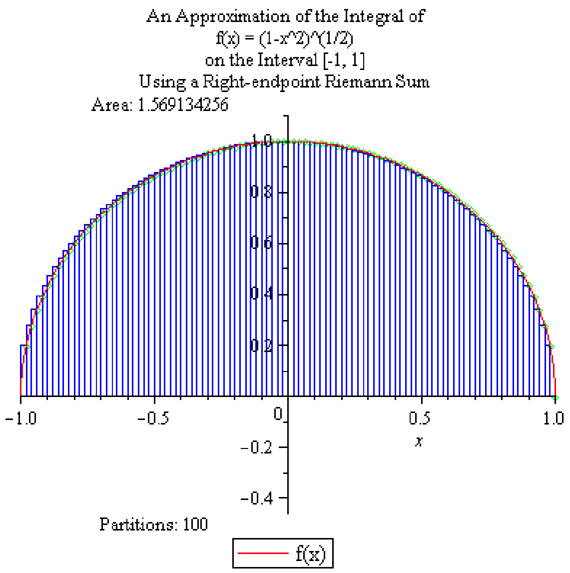
$$x_k = -1 + k\cdot\frac{1}{2} = a + k\Delta x$$

$$= -1 + \frac{k}{2}$$

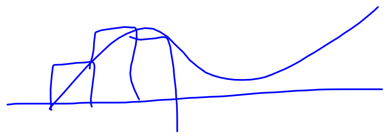

An Approximation of the Integral of
f(x) = (1-x^2)^(1/2)
on the Interval [-1, 1]
Using a Right-endpoint Riemann Sum
Area: 1.518524415
1.0
0.8
0.6
0.4
0.2
-1.0
-0.5
0
0.5
1.0
x
-0.2
-0.4
Partitions: 10
f(x)


An Approximation of the Integral of
f(x) = (1-x^2)^(1/2)
on the Interval [-1, 1]
Using a Right-endpoint Riemann Sum
Area: 1.552259162
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-1.0
-0.5
0.5
1.0
x
Partitions: 20
f(x)


An Approximation of the Integral of
f(x) = (1-x^2)^(1/2)
on the Interval [-1, 1]
Using a Right-endpoint Riemann Sum
Area: 1.566098155
1.0
0.8
0.6
0.4
0.2
-1.0
-0.5
0
0.5
1.0
x
-0.2
-0.4
Partitions: 50
f(x)


An Approximation of the Integral of
f(x) = (1-x^2)^(1/2)
on the Interval [-1, 1]
Using a Right-endpoint Riemann Sum
Area: 1.569134256
1.0
0.8
0.6
0.4
0.2
-1.0
-0.5
0
0.5
1.0
x
-0.2
-0.4
Partitions: 100
f(x)

1.570796327



Coming Up: Calculus!

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x = \text{limit of Riemann Sum}$$

$$= \int_a^b f(x)\,dx = \text{EXACT AREA.}$$