Section 4.4 Indefinite Integrals and Net Change Theorem

$\int cf(x)\,dx = c\int f(x)\,dx$

$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

$\int k\,dx = kx + C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

$\int \sin x\,dx = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \csc x \cot x\,dx = -\csc x + C$

Integral is a linear operator, i.e., it respects addition and scalar multiplication.

I'm not sure I'd bother remembering these trig antiderivatives. They come from what we already know about derivatives, and we can work backwards from derivatives we know and that I'll give you.

Net Change Theorem The integral of a rate of change is the net change:

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

1. Question Details SCalc8 4.4.001

State whether the following is true or false by differentiation.

$$\int \frac{9}{x^2\sqrt{1+x^2}}\,dx = -\frac{9\sqrt{1+x^2}}{x} + C = -\frac{9(x^2+1)^{\frac{1}{2}}}{x} + C = f(x) \Rightarrow$$

$$f'(x) = -9\left[\frac{\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)x - (x^2+1)^{\frac{1}{2}}(1)}{x^2}\right]$$

$$= -9\left[\frac{\frac{x^2}{\sqrt{x^2+1}} - \frac{\sqrt{x^2+1}}{1}\cdot\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}}{x^2}\right] = -9\left[\frac{\frac{x^2-(x^2+1)}{\sqrt{x^2+1}}}{x^2}\right] = -9\left[\frac{-1}{x^2\sqrt{x^2+1}}\right]$$

$$= \frac{9}{x^2\sqrt{x^2+1}}.$$ So, yes!

True!

2. Question Details SCalc8 4.4.002.

State whether the following is true or false by differentiation.

$$\int \cos^2(x)\,dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C = f(x) \Rightarrow$$

$$f'(x) = \frac{1}{2} + \frac{1}{4}(\cos(2x))(2) = \frac{1}{2} + \frac{1}{2}\cos(2x) = \frac{1}{2} + \frac{1}{2}\left[2\cos^2 x - 1\right]$$

$$= \frac{1}{2} + \cos^2 x - \frac{1}{2} = \cos^2 x$$

$$\int \cos^2 x\,dx = \int \frac{\cos(2x)+1}{2}\,dx = \frac{1}{2}\int(\cos(2x)+1)\,dx$$

Don't know how to do chain rule in reverse, for the "2x" inside.

3. Question Details SCalc8 4.4.005.

Find the general indefinite integral. (Use C for the constant of integration.)

$$\int (x^{1.4} + 9x^{3.5})\,dx$$

$1.4 + 1 = 2.4$

$3.5 + 1 = 4.5$

$$= \frac{x^{2.4}}{2.4} + 9\left(\frac{x^{4.5}}{4.5}\right) + C$$

$$= \boxed{\frac{1}{2.4}x^{2.4} + 2x^{4.5} + C}$$

4. Question Details SCalc8 4.4.006.

Find the general indefinite integral. (Use C for the constant of integration.)

$$\int \sqrt[8]{x^9}\,dx = \int x^{\frac{9}{8}}\,dx = \frac{x^{17/8}}{\frac{17}{8}} + C = \boxed{\frac{8x^{\frac{17}{8}}}{17} + C}$$

5. Question Details SCalc8 4.4.008.

Find the general indefinite integral. (Use C for the constant of integration.)

$$\int \left(u^7 - 3u^6 - u^4 + \frac{6}{3}\right) du$$

$$= \boxed{\frac{1}{8}u^8 - \frac{3}{7}u^7 - \frac{1}{5}u^5 + 2u + C}$$

6. Question Details SCalc8 4.4.009.

Find the general indefinite integral. (Use C for the constant of integration.)

$$\int (u + 1)(2u + 7)\, du = \int (2u^2 + 9u + 7)\, du = \boxed{\frac{2}{3}u^3 + \frac{9}{2}u^2 + 7u + C}$$

7. Question Details SCalc8 4.4.01

Find the general indefinite integral. (Use C for the constant of integration.)

$$\int \sec(t)(8 \sec(t) + 9 \tan(t))\, dt = \int \left(8 \sec^2(t) + 9 \sec(t) \tan(t)\right) dt$$

$$= \boxed{8 \tan(t) + 9 \sec(t) + C}$$

Use Table, or work derivatives table, backwards.

$\sec^2(t) = \frac{d}{dt} \tan(t)$

$\sec(t)\tan(t) = \frac{d}{dt} \sec(t)$

8. Question Details SCalc8 4.4.016.

Find the general indefinite integral. (Use C for the constant of integration.)

$$\int 7\frac{\sin(2x)}{\sin(x)}\, dx = 7 \int \frac{2\sin(x)\cos(x)}{\sin(x)}\, dx = 14 \int \cos(x)\, dx = \boxed{14 \sin(x) + C}$$

$\frac{d}{dx} \sin(x) = \cos x$

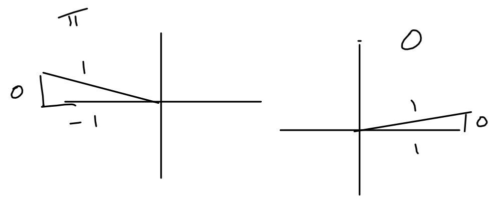
9. Question Details SCalc8 4.4.025.

Evaluate the integral.

$$\int_0^{\pi} (3 \sin \theta - 11 \cos \theta)\, d\theta = 3 \int_0^{\pi} \sin\theta \, d\theta - 11 \int_0^{\pi} \cos\theta \, d\theta$$

$$= \Big[3(-\cos\theta) - 11 \sin\theta\Big]_0^{\pi} = -3\cos\pi - 11 \sin\pi - \Big[-3\cos(0) - 11\sin(0)\Big]$$

$$= -3(-1) - 11(0) - \Big[-3(1) - 11(0)\Big] = 3 - (-3) = \boxed{6}$$



10. Question Details SCalc8 4.4.026

Evaluate the integral.

$$\int_1^4 \left(\frac{1}{x^2} - \frac{8}{x^3}\right) dx$$

Tell me in class, tomorrow, what $\int \frac{1}{x}\, dx$ is !

$$= \int_1^4 (x^{-2} - 8x^{-3})\, dx = \left[\frac{x^{-1}}{-1} - 8\left(\frac{x^{-2}}{-2}\right)\right]_1^4$$

$$= \left[-\frac{1}{x} + \frac{4}{x^2}\right]_1^4 = -\frac{1}{4} + \frac{4}{4^2} - \left[-\frac{1}{1} + \frac{4}{1^2}\right] = -\frac{1}{4} + \frac{1}{4} - \left[-1 + 4\right] = -\left[3\right] = \boxed{-3}$$

11. Question Details SCalc8 4.4.029.

Evaluate the integral.

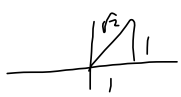
$$\int_1^9 \sqrt{\frac{5}{x}}\, dx = \int_1^9 \frac{\sqrt{5}}{\sqrt{x}}\, dx = \int_1^9 \sqrt{5}\, x^{-\frac{1}{2}}\, dx = \sqrt{5}\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^9$$

$$= 2\sqrt{5}\left[x^{\frac{1}{2}}\right]_1^9 = 2\sqrt{5}\left[9^{\frac{1}{2}} - 1^{\frac{1}{2}}\right] = 2\sqrt{5}\left[3 - 1\right] = \boxed{4\sqrt{5}}$$

12. Question Details SCalc8 4.4.033.

Evaluate the integral.

$$\int_0^{\pi/4} \frac{4 + 5\cos^2(\theta)}{\cos^2(\theta)}\, d\theta = \int_0^{\frac{\pi}{4}} \left(\frac{4}{\cos^2\theta} + \frac{5\cos^2\theta}{\cos^2\theta}\right) d\theta$$



$$= \int_0^{\frac{\pi}{4}} (4\sec^2\theta + 5)\, d\theta = \Big[4\tan\theta + 5\theta\Big]_0^{\frac{\pi}{4}} = 4\tan\left(\frac{\pi}{4}\right) + 5\left(\frac{\pi}{4}\right) - \Big[4\tan(0) + 5(0)\Big]$$

$$= 4(1) + \frac{5\pi}{4} - \Big[0 + 0\Big] = \boxed{4 + \frac{5\pi}{4}}$$

13. Question Details SCalc8 4.4.034.

Evaluate the integral.

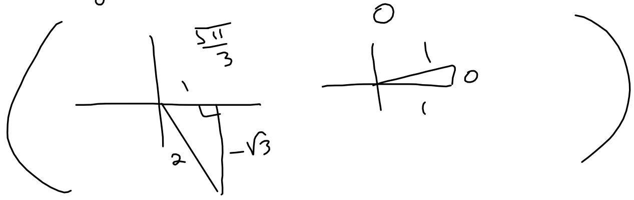
$$\int_0^{5\pi/3} \frac{4\sin(\theta)+4\sin(\theta)\tan^2(\theta)}{\sec^2(\theta)}\,d\theta$$

Alternate: $\frac{4\sin\theta\left[1+\tan^2\theta\right]}{\sec^2\theta} = \frac{4\sin\theta\left[\sec^2\theta\right]}{\sec^2\theta} = 4\sin\theta$

$$\frac{4\sin\theta}{\sec^2\theta}+\frac{4\sin\theta\tan^2\theta}{\sec^2\theta} = 4\sin\theta\underbrace{\cos^2\theta}_{1-\sin^2\theta} + 4\sin\theta\underbrace{\tan^2\theta\cos^2\theta}_{\sin^2\theta}$$

$$= 4\sin\theta - 4\sin^3\theta + 4\sin^3\theta = 4\sin\theta$$ cool!

$$= \int_0^{\frac{5\pi}{3}} 4\sin\theta\,d\theta = 4(-\cos\theta)\Big]_0^{\frac{5\pi}{3}} = -4\cos\frac{5\pi}{3} - (-4\cos(0))$$



$$= -4\left(\tfrac{1}{2}\right) + 4(1) = -2+4 = \boxed{2}$$

14. Question Details SCalc8 4.4.041.

Evaluate the integral.

$$\int_{-2}^{1} (x - 4|x|)\,dx$$

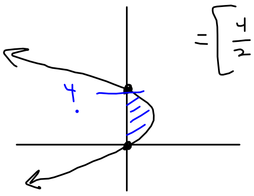
$$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$= \int_{-2}^{0} (x-4(-x))\,dx + \int_0^1 (x-4x)\,dx = \int_{-2}^{0} 5x\,dx + \int_0^1 -3x\,dx = \frac{5x^2}{2}\Big]_{-2}^{0} - \left[\frac{3x^2}{2}\right]_0^1$$

$$= \underbrace{\frac{5(0)^2}{2}}_{0} - \frac{5(-2)^2}{2} - \left[\frac{3(1)^2}{2} - \underbrace{\frac{3(0)^2}{2}}_{0}\right] = -\frac{20}{2} - \left[\frac{3}{2}\right] = \boxed{\frac{17}{2}}$$

15. Question Details SCalc8 4.4.045. [3353673]

The area of the region that lies to the right of the y-axis and to the left of the parabola $x = 4y - y^2$ (the shaded region in the figure) is given by the integral $\int_0^4 (4y - y^2)\, dy$. (Turn your head clockwise and think of the region as lying below the curve $x = 4y - y^2$ from $y = 0$ to $y = 4$.) Find the area of the region.



$$= \left[\frac{4}{2}y^2 - \frac{1}{3}y^3\right]_0^4$$

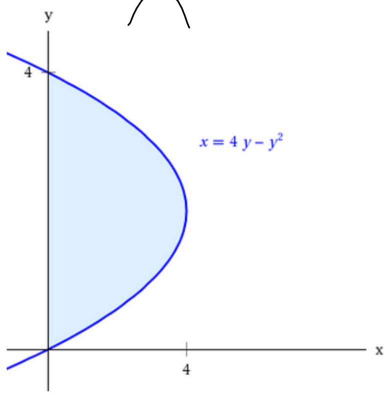
$$= 2(4)^2 - \frac{1}{3}(4)^3 - [0-0]$$

$$= 32 - \frac{64}{3} = \frac{96-64}{3} = \boxed{\frac{32}{3}}$$

$\int_0^4 (4x - x^2)\, dx$ = same thing.

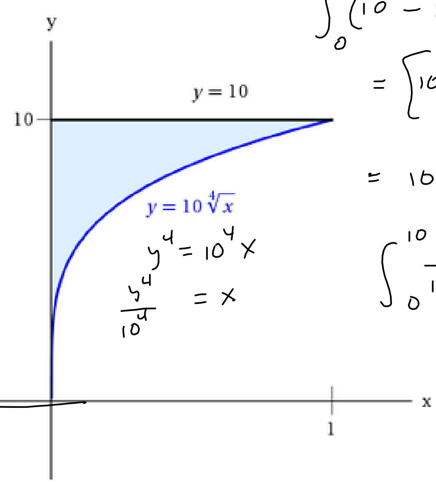
$-y^2 + 4y = -y(y-4)$

Pic. from Book.



16. Question Details SCalc8 4.4.046. [3395341]

The boundaries of the shaded region in the figure are the y-axis, the line $y = 10$, and the curve $y = 10\sqrt[4]{x}$. Find the area of this region by writing x as a function of y and integrating with respect to y.



$y =$ 10

$y = 0$

$y^4 = 10^4 x$

$\frac{y^4}{10^4} = x$

$$\int_0^1 (10 - 10\sqrt[4]{x})\, dx = \int_0^1 (10 - 10x^{\frac{1}{4}})\, dx \qquad 10\left(\tfrac{4}{5}\right) = 8$$

$$= \left[10x - 10\left(\frac{x^{\frac{5}{4}}}{\frac{5}{4}}\right)\right]_0^1 = \left[10x - 8x^{\frac{5}{4}}\right]_0^1$$

$$= 10 - 8 = \boxed{2}$$

$$\int_0^{10} \frac{1}{10^4} y^4\, dy = \frac{1}{10^4} \cdot \frac{y^5}{5}\Big]_0^{10} = \frac{10^5}{10^4 \cdot 5} = \frac{10}{5} = \boxed{2}$$

17. Question Details SCalc8 4.4.047.

If $w'(t)$ is the rate of growth of a child in pounds per year, what does $\int_5^{13} w'(t)dt$ represent?

The net amount of weight gained from age 5 to age 13.

18. Question Details SCalc8 4.4.048. [3353948]

A current exists whenever electric charges move. If ΔQ is the net charge that passes through a surface during a time period Δt, then the average current during this time interval is defined as

$$\text{average current} = \frac{\Delta Q}{\Delta t} = \frac{Q_2 - Q_1}{t_2 - t_1}.$$

If we take the limit of this average current over smaller and smaller time intervals, we get what is called the **current** I at a given time t_1:

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}.$$

Thus the current is the rate at which charge flows through a surface.

The current in a wire is defined as the derivative of the charge: $\boxed{I(t) = Q'(t). = \frac{dQ}{dt}}$ What does $\int_a^b I(t)\,dt$ represent?

The net change in the charge.

19. Question Details SCalc8 4.4.052. [3353615]

If $f(x)$ is the slope of a trail at a distance of x miles from the start of the trail, what does $\int_3^7 f(x)dx$ represent?

The net change in elevation from x = 3 to x = 7 miles (map miles) along the trail.

20. Question Details SCalc8 4.4.059. [33539

The linear density of a rod of length 4 m is given by $\rho(x) = 7 + 7\sqrt{x}$ measured in kilograms per meter, where x is measured in meters from one end of the rod. Find the total mass of the rod.

Density is like a rate of change in mass, per unit length (or small increment) of length along the rod.

$$\int_0^4 (7 + 7x^{\frac{1}{2}})dx = \left[7x + \frac{14}{3}x^{\frac{3}{2}}\right]_0^4 = 7(4) + \frac{14}{3}(4)^{\frac{3}{2}} - [0+0]$$

$$= 28 + \frac{14}{3}(8) = 28 + \frac{112}{3} = \frac{84 + 112}{3} = \boxed{\frac{196}{3}\text{ kg}}$$

$$4^{\frac{3}{2}} = (4^3)^{\frac{1}{2}} = (4^{\frac{1}{2}})^3 = 2^3 = 8$$

21. Question Details SCalc8 4.4.070.

Evaluate the integral.

$$\int_{-11}^{11} \frac{3e^x}{\sinh(x) + \cosh(x)}\,dx$$

Balanced integral.

http://webhome.phy.duke.edu/~rgb/Class/phy51/phy51/node15.html

$$\sinh(x) = \frac{e^x - e^{-x}}{2}, \quad \cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}, \quad \cos(x) = \frac{e^{ix} + e^{-ix}}{2}$$

$$\frac{3e^x}{\left(\frac{e^x - e^{-x} + e^x + e^{-x}}{2}\right)} = \frac{6e^x}{2e^x} = 3$$

$$\int_{-11}^{11} 3\,dx = 2\int_0^{11} 3\,dx = 6\left[x\right]_0^{11} = \boxed{66}$$

3 is even