Section 4.3 Fundamental Theorem of Calculus

FTC I f cnt^s on $[a,b]$, $a < x < b \implies$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\frac{d}{dx}\int_0^x \frac{\sin(t)\cos(t)}{t^2+1}\,dt = \frac{\sin(x)\cos(x)}{x^2+1}$$

Chain Rule Version

$$\frac{d}{dx}\int_a^{u(x)} f(t)\,dt = f(u(x))u'(x)$$

$$\frac{d}{dx}\int_0^{x^2-x} \frac{\sin(t)\cos(t)}{t^2+1}\,dt = \left(\frac{\sin(x^2-x)\cos(x^2-x)}{(x^2-x)^2+1}\right)(2x-1)$$

"Proof"

$$g(x) = \int_a^x f(t)dt \implies \frac{g(x+h)-g(x)}{h} = \frac{\int_a^{x+h} f(t)dt - \int_a^x f(t)dt}{h}$$

$$= \frac{1}{h}\left[\int_a^x f(t)dt + \int_x^{x+h} f(t)dt - \int_a^x f(t)dt\right] = \frac{1}{h}\left[\int_x^{x+h} f(t)\,dt\right]$$

f cnt^s on $[a,b]$ & assuming x & $x+h \in [a,b]$

Then EVT says $\exists\ u \in [x, x+h]$ such that $f(u)$ is min on $[x, x+h]$ & $v \in [x,x+h] \ni f(v)$ is max on $[x, x+h]$.

Then $f(u)h \le \int_x^{x+h} f(t)dt \le f(v)h$ Assume $h > 0$.

$$f(u) \le \frac{\int_x^{x+h} f(t)\,dt}{h} \le f(v)$$

f is cnt^s $\implies \lim_{h\to 0} f(x+h) = f(x)$

& u & v both live in $[x, x+h]$. So as $h \to 0$, $f(u) \to f(x)$, $f(v) \to f(x)$

by Squeeze Thm, passing to the limit

$$f(x) \le \frac{d}{dx}\int_x^{x+h} f(t)\,dt \le f(x)$$

$$f(x) = \frac{d}{dx}\int_x^{x+h} f(t)dt$$

FTC II

f cnt^s on $[a,b]$ and F be any antiderivative of f.

$$\Longrightarrow \int_a^b f(t)\,dt = F(b) - F(a)$$

$g(x) = \int_a^x f(t)\,dt$. Then $g'(x) = f(x)$, i.e., g is an antiderivative, so $F(x) = g(x) + C$. F & g are cnt^s on (a,b) & some limits as $x \to a^+$ & $x \to b^-$ give us on $[a,b]$. $F(x) = g(x) + C$ on $[a,b]$

$$g(a) = \int_a^a f(t)\,dt = 0$$

$$F(b) - F(a) = [g(b) + C] - [g(a) + C] = g(b) - g(a)$$

$$= g(b) = \int_a^b f(t)\,dt.$$

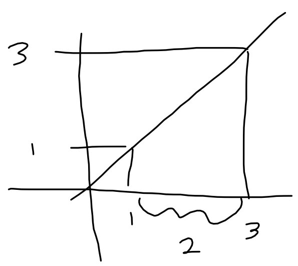
$$\int_1^3 x\,dx = \left.\frac{x^2}{2}\right]_1^3 = \frac{3^2}{2} - \frac{1^2}{2} = \frac{9-1}{2} = \frac{8}{2} = 4$$

$F(b) - F(a)$

We always choose $F \ni C = 0$.

$\frac{1}{2}(1+3)(2) = 4$

$\frac{1}{2}(b_1 + b_2)h$



1. Question Details SCalc8 4.3.002.

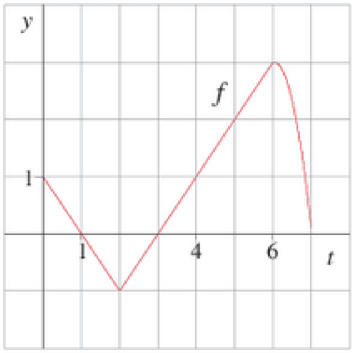
Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.

(a) Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4, 5$, and 6.

(b) Estimate $g(7)$. (Use the midpoint to get the most precise estimate.)

(c) Where does g have a maximum and a minimum value?

(d) Sketch a rough graph of g.



2. Question Details SCalc8 4.3.005.

Sketch the area represented by $g(x)$.

$$g(x) = \int_1^x t^2\,dt$$

Then find $g'(x)$ in two of the following ways.

(a) by using Part 1 of the Fundamental Theorem

(b) by evaluating the integral using Part 2 and then differentiating

3. Question Details SCalc8 4.3.007.

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$g(x) = \int_0^x \sqrt{t + t^3}\, dt \implies \boxed{g'(x) = \sqrt{x + x^3}}$$

4. Question Details

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$F(x) = \int_x^0 \sqrt{4 + \sec(9t)}\, dt \quad \left[\textit{Hint: } \int_x^0 \sqrt{4 + \sec(9t)}\, dt = -\int_0^x \sqrt{4 + \sec(9t)}\, dt\right]$$

$$= -\int_0^x \sqrt{4 + \sec(9t)}\, dt = -\sqrt{4 + \sec(9x)}$$

5. Question Details SCalc8 4.3.019.

Evaluate the integral.

$$\int_4^6 (x^2 + 2x - 2)\, dx$$

6. Question Details SCalc8 4.3.023.

Evaluate the integral.

$$\int_{4}^{9} \sqrt{x}\, dx$$

7. Question Details SCalc8 4.3.025.

Evaluate the integral.

$$\int_{\pi/6}^{\pi} \sin(\theta)\, d\theta$$

8. Question Details SCalc8 4.3.027.

Evaluate the integral.

$$\int_{0}^{1} (u + 6)(u - 7)\, du$$

9. Question Details SCalc8 4.3.029.

Evaluate the integral.

$$\int_{1}^{9} \frac{4 + x^2}{\sqrt{x}}\, dx$$

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10. Question Details SCalc8 4.3.031

Evaluate the integral.

$$\int_{\pi/6}^{\pi/3} \csc(t)\cot(t)\,dt$$

11. Question Details SCalc8 4.3.038.

Evaluate the integral.

$$\int_{-3}^{2} f(x)\,dx \text{ where } f(x) = \begin{cases} 4 & \text{if } -3 \le x \le 0 \\ 6 - x^2 & \text{if } 0 < x \le 2 \end{cases}$$

12. Question Details SCalc8 4.3.039.

Sketch the region enclosed by the given curves. (A graphing calculator is recommended.)

$y = \sqrt{x}$, $y = 0$, $x = 4$ Calculate its area.

13. Question Details SCalc8 4.3.041.

Sketch the region enclosed by the given curves. (A graphing calculator is recommended.)

$y = 4 - x^2$, $y = 0$ Calculate its area.

14. Question Details SCalc8 4.3.043. [3353884]

Use a graph to give a rough estimate of the area of the region that lies beneath the given curve. Then find the exact area.

$y = \sqrt[4]{x}, \quad 0 \le x \le 81$

15. Question Details SCalc8 4.3.045. [3353768]

Use a graph to give a rough estimate of the area of the region that lies beneath the given curve. Then find the exact area.

$y = 4\sin(x), \quad 0 \le x \le \pi$

16. Question Details SCalc8 4.3.046. [3353599]

Use a graph to give a rough estimate of the area of the region that lies beneath the given curve. Then find the exact area.

$y = \sec^2(x), \quad 0 \le x \le \pi/6$

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17. Question Details SCalc8 4.3.047.

Evaluate the integral and interpret it as a difference of areas.

$$\int_{-1}^{2} x^3\,dx$$

Illustrate with a sketch.

18. Question Details SCalc8 4.3.049.

What is wrong with the equation?

$$\int_{-4}^{2} x^{-3}\,dx = \left[\frac{x^{-2}}{-2}\right]_{-4}^{2} = -\frac{3}{32}$$

19. Question Details SCalc8 4.3.053.

Find the derivative of the function.

$$g(x) = \int_{3x}^{5x} \frac{u^2-4}{u^2+4}\,du \quad \left[\text{Hint: } \int_{3x}^{5x} f(u)\,du = \int_{3x}^{0} f(u)\,du + \int_{0}^{5x} f(u)\,du\right]$$

20. Question Details SCalc8 4.3.056.

Find the derivative of the function.

$$g(x) = \int_{\tan x}^{4x^2} \frac{1}{\sqrt{3+t^3}}\,dt = \int_{\tan x}^{0} \sim + \int_{0}^{4x^2}$$

$$= -\int_{0}^{\tan(x)} \frac{1}{\sqrt{3+t^3}}\,dt + \int_{0}^{4x^2} \frac{1}{\sqrt{3+t^3}}\,dt \longrightarrow g'(x) =$$

$$-\left(\frac{1}{\sqrt{3+\tan^3(x)}}\right)\left(\sec^2(x)\right) + \frac{1}{\sqrt{3+(4x^2)^3}}\,(8x)$$

21. Question Details SCalc8 4.3.06

If $f(1) = 13$, f' is continuous, and $\int_1^5 f'(x)\,dx = 18$, what is the value of $f(5)$?

22. Question Details SCalc8 4.3.065.

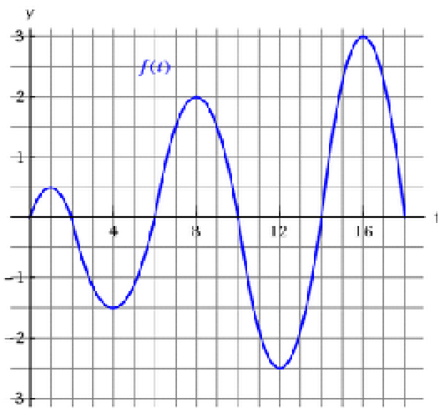
Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.

(a) At what values of x do the local maximum and minimum values of g occur?

(b) Where does g attain its absolute maximum value?

(c) On what interval is g concave downward? (Enter your answer using interval notation.)

(d) Sketch the graph of g.



23. Question Details SCalc8 4.3.071.

(a) Show that $1 \le \sqrt{1 + x^3} \le 1 + x^3$ for $x \ge 0$.

(b) Show that $1 \le \int_0^1 \sqrt{1 + x^3}\, dx \le 1.25$.

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