Section 4.1 Distances and Areas

Rate times Time = Distance
Height times Width = Area

} We'll be representing Distance as the area under a Rate function:

$y = f(t) =$ rate function, in $\frac{\text{miles}}{\text{hour}}$, as a function of

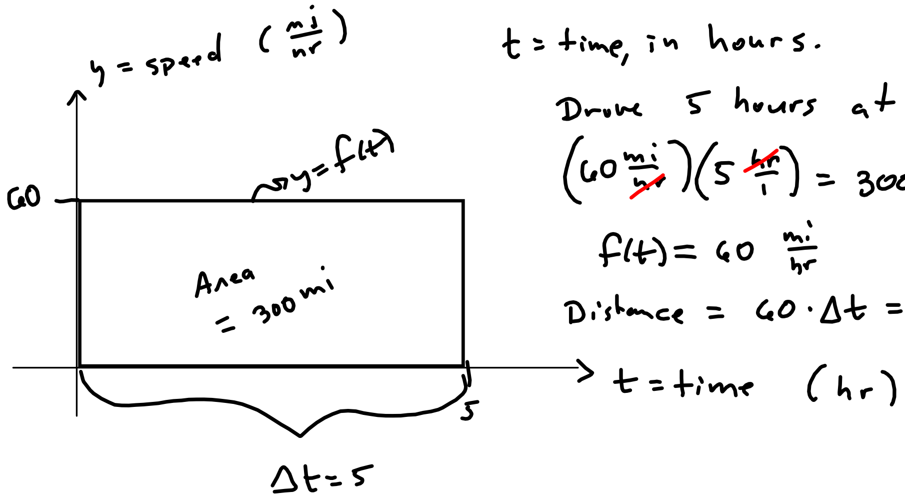
$t =$ time, in hours.

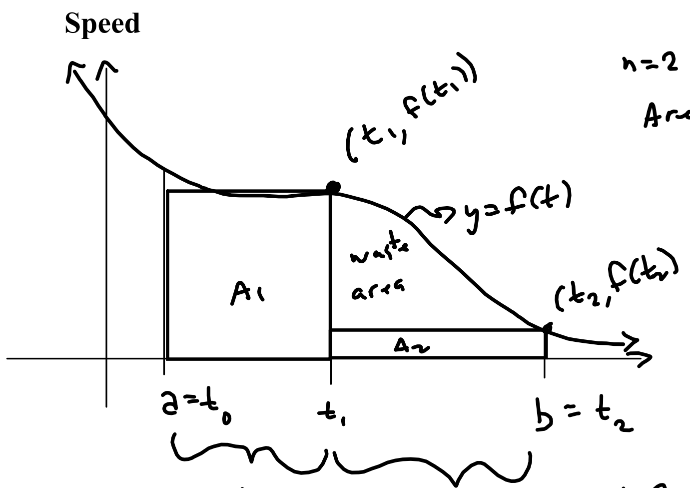
Drove 5 hours at $60 \frac{mi}{hr}$

$\left(60 \frac{mi}{\cancel{hr}}\right)\left(5 \frac{\cancel{hr}}{1}\right) = 300$ mi

$f(t) = 60 \frac{mi}{hr}$

Distance $= 60 \cdot \Delta t = f(t)\Delta t$





$n = 2$ rectangles.

Area =

Area under curve is approximately

$A_1 + A_2 = h_1 \cdot w_1 + h_2 \cdot w_2$

$= f(t_1)(t_1 - t_0) + f(t_2)(t_2 - t_1)$

Let $\Delta t = t_1 - t_0 = t_2 - t_1$

$= \frac{b-a}{2}$

$\Delta t = t_1 - t_0 = \frac{b-a}{n} = \frac{b-a}{2}$

$\Delta t = w_2 = t_2 - t_1 = \frac{b-a}{n} = \frac{b-a}{2}$

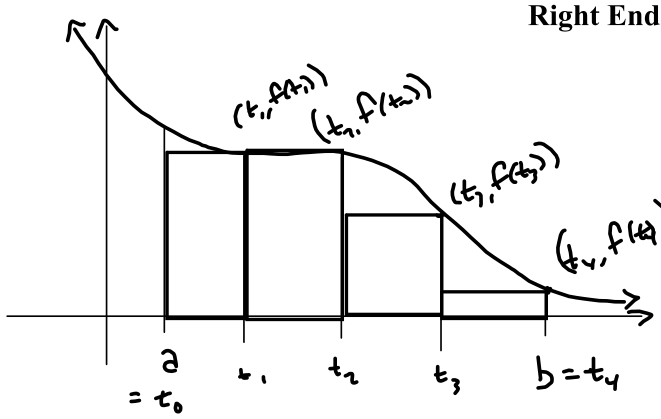
Use same widths $\Delta t = \frac{b-a}{2}$

Area $\approx$ Sum of areas of the rectangles

$f(t_1)\Delta t + f(t_2)\Delta t = \Delta t \left(f(t_1) + f(t_2)\right)$

$= \sum_{k=1}^{2} f(t_k)\Delta t = \Delta t \sum_{k=1}^{2} f(t_k)$

Right Endpoints, n = 4.



$$\text{Area} \approx \sum_{k=1}^{4} f(t_k)\Delta t$$

$$t_1 = t_0 + \Delta t$$
$$t_2 = t_1 + \Delta t = t_0 + 2\Delta t$$
$$= t_0 + \Delta t + \Delta t$$
$$\vdots$$
$$t_k = t_0 + k\Delta t$$

If the widths are all the same width, then

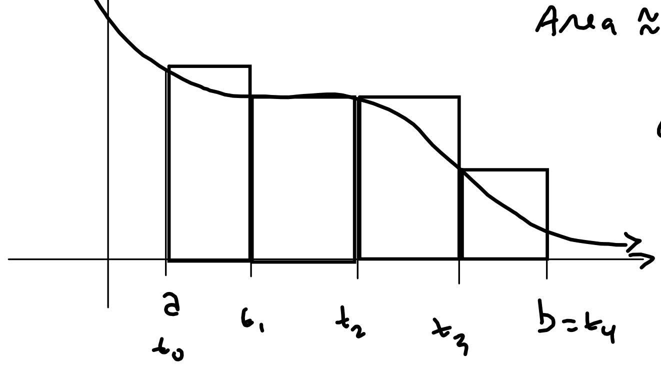
$$\Delta t = \frac{b-a}{n} = \frac{b-a}{4}$$

$$t_k = t_0 + k\Delta t$$
$$= t_0 + k\left(\frac{b-a}{n}\right)$$

$$\text{Area} \approx \frac{b-a}{4} \sum_{k=1}^{4} f(t_k)$$

$$= \frac{b-a}{4} \sum_{k=1}^{4} f\left(t_0 + k\frac{b-a}{n}\right)$$

Left Endpoints Riemann Sum.



$$\text{Area} \approx \frac{b-a}{4} \sum_{k=1}^{4} f\left(t_0 + (k-1)\frac{b-a}{n}\right)$$

Other method:
Midpoint Rule.