

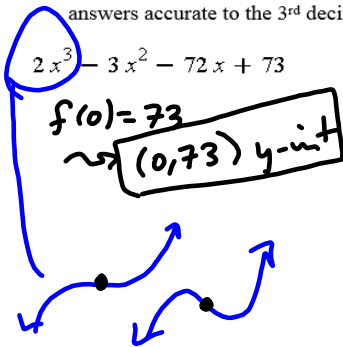
MAT 201
100 Points

Covers Chapter 3

Test 2 - Fall, 2013

Name _____
NO GRAPHING CALCULATORS!!!

1. (10 pts) Let $f(x) = x^3 - 3x^2 + 2x$. Find all absolute and local extremes of f on the interval $[0, 3]$. Final answers accurate to the 3rd decimal place are acceptable.



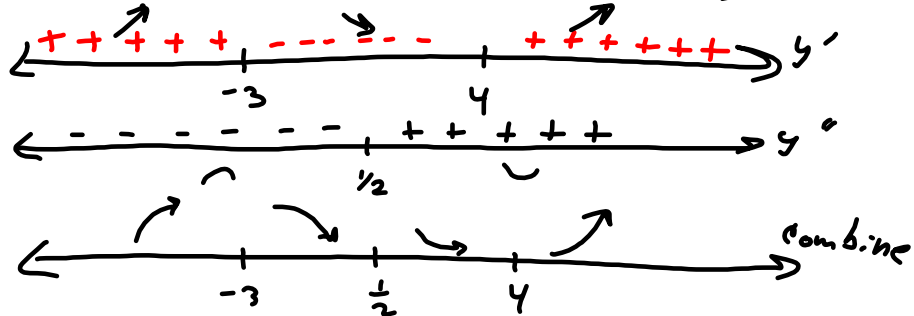
Make $f'(x) = 0$ nice
 Make either $f(1)$ or $f(2)$ or $f(3) = 0$
 to make intercepts nice/double.
 Emphasis not on x-int. (Bonus).

$$f'(x) = 6x^2 - 6x - 72 = 6(x^2 - x - 12)$$

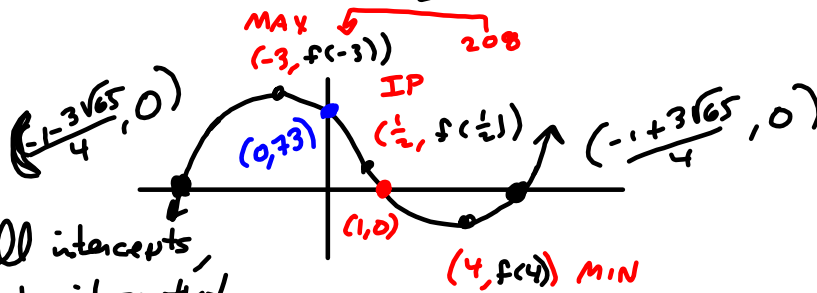
$$= 6(x-4)(x+3) \text{ SET } 0 \Rightarrow$$

$$x \in \{-3, 4\}$$

$$f''(x) = 6[2x-1] \text{ SET } 0 \Rightarrow x \in \{\frac{1}{2}\}$$



GRAPH
Video #1



Find all intercepts,
 Steve made it so that
 $f(1)/f(2)/f(3)$ is zero.

$$2x^3 - 3x^2 - 72x + 73$$

Guess $x = 1$:

$$\begin{array}{r} \underline{1) 2 \quad -3 \quad -72 \quad 73} \\ \quad 2 \quad -1 \quad -73 \\ \hline 2 \quad -1 \quad -73 \quad 0 \end{array}$$

$$\begin{array}{r} 273 \\ 8 \\ \hline 584 \end{array}$$

$$(x-1)(2x^2 - x - 73)$$

$$a=2, b=1, c=-73$$

$$b^2 - 4ac = (-1)^2 - 4(2)(-73)$$

$$= 1 + 584$$

$$= 585 = 3^2 \cdot 5 \cdot 13$$

$$\Rightarrow \sqrt{585} = 3\sqrt{65}$$

$$\begin{array}{r} 3 \overline{) 585} \\ \underline{3} \\ 195 \\ \underline{3} \\ 65 \\ \underline{5} \\ 13 \end{array}$$

$f(-3)$:

$$\begin{array}{r} \underline{-3) 2 \quad -3 \quad -72 \quad 73} \\ \quad -6 \quad 27 \quad 135 \\ \hline 2 \quad -9 \quad -45 \quad 208 \end{array}$$

$(-3, 208)$

$$x = \frac{-1 \pm 3\sqrt{65}}{2(2)} = \frac{-1 \pm 3\sqrt{65}}{4}$$

2. (10 pts) Confirm that $f(x) = x^3 - 3x^2 + 2x$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 3]$. Then find all values c in $(0, 3)$ that satisfy the conclusion of the theorem.

Maximize $f(x) = 2x^3 - 3x^2 - 72x + 73$ on $[0, 3]$

Find f' , SET $f' = 0$, etc.

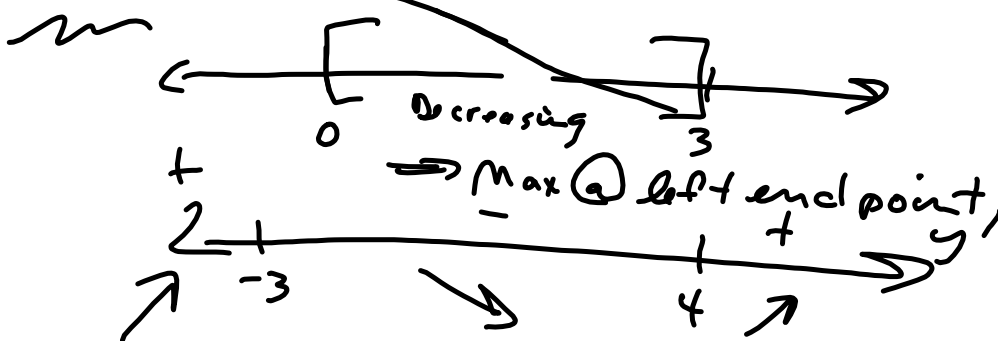
$$f'(x) = 6x^2 - 6x - 72$$

$$\begin{array}{r} 2 \overline{) 2 \quad -3 \quad -72 \quad 73} \\ \underline{6 \quad 9 \quad -189} \\ 2 \quad 3 \quad -63 \quad -116 \end{array}$$

$$\begin{array}{r} 63 \\ 3 \\ \hline -189 \\ \hline 73 \\ \hline -116 \end{array}$$

$f(0) = 73 \rightarrow (0, 73)$
 $f(3) = -116 \quad \text{MAX}$

$f'(x) = 0 \text{ @ } x = -3, x = 4$



MAXIMIZE
 FUNCTION ON A
 CLOSED
 INTERVAL
 Video #2

MEAN VALUE
THEOREM
QUESTION.
Video #3

$$x^3 - 3x^2 + 2x$$

$$f(0) = 0$$

$$f(3) = 6$$

$$m_{AVG} = \frac{f(3) - f(0)}{3 - 0}$$

$$= \frac{6 - 0}{3} = 2 = m_{AVG}$$

$$f'(x) = 3x^2 - 6x + 2 \stackrel{SET}{=} 2 = m_{avg} \quad \exists c \in (0, 3) \Rightarrow$$

$$\Rightarrow 3x^2 - 6x = 0$$

$$\Rightarrow 3x(x - 2) = 0$$

$$\Rightarrow x \in \{0, 2\}$$

$$\boxed{c = 2 \in (0, 3)}$$

MVT hypotheses on $[0, 3]$?

$\left. \begin{array}{l} \text{cont}^d \text{ on } [0, 3] \\ \text{diff}^d \text{ on } (0, 3) \end{array} \right\} \begin{array}{l} f \text{ is polynomial} \\ \Rightarrow \\ \text{diff}^d \text{ \& } \text{cont}^d \\ \forall x \in \mathbb{R} \end{array}$

MVT conclusion is

$$f'(c) = m_{avg} \text{ on } [0, 3]$$

3. (10 pts) Let $f(x) = -2\sin(x)\cos(x) - x$. Find all local extrema in the interval $[0, 2\pi]$.

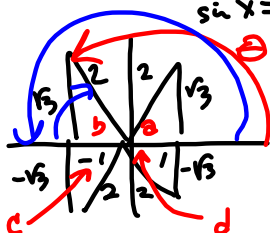
Not endpoints!

$$\begin{aligned} f'(x) &= -2[\cos^2 x - \sin^2 x] - 1 \\ &= -2[1 - \sin^2 x - \sin^2 x] - 1 \\ &= -2[1 - 2\sin^2 x] - 1 \\ &= -2 + 4\sin^2 x - 1 \\ &= 4\sin^2 x - 3 \stackrel{SET}{=} 0 \end{aligned}$$

$4u^2 - 3 = 0, \text{ etc.}$

$\rightarrow \sin^2 x = \frac{3}{4}$
 $\sin x = \pm \frac{\sqrt{3}}{2}$

$(2\sin x - \sqrt{3})(2\sin x + \sqrt{3})$

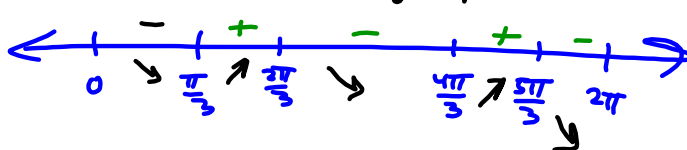


$\alpha = 60^\circ = \frac{\pi}{3}$
 $b = 60^\circ$ is reference
 so $\theta = 180^\circ - 60^\circ$
 $= 120^\circ$
 $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

critical values in $(0, 2\pi)$

Sign pattern

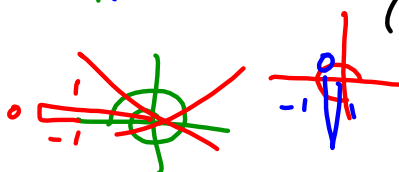


$f'(\frac{\pi}{6})$
 $= 4\sin^2(\frac{\pi}{6}) - 3$
 $= 4(\frac{1}{2})^2 - 3$
 $= 1 - 3 < 0$



$\frac{4\pi}{3} + \frac{5\pi}{3} = \frac{9\pi}{3} = \frac{3\pi}{2}$
 $4\sin^2(\frac{3\pi}{2}) - 3$
 $= 4(-1)^2 - 3 = 1$

- $(\frac{\pi}{3}, _)$ min
- $(\frac{2\pi}{3}, _)$ Max
- $(\frac{4\pi}{3}, _)$ min
- $(\frac{5\pi}{3}, _)$ Max



$evalf(f(0))$	0.
$evalf(f(\frac{\pi}{3}))$	-1.913222955
$evalf(f(\frac{2\pi}{3}))$	-1.228369699
$evalf(f(\frac{4\pi}{3}))$	-5.054815608
$evalf(f(\frac{5\pi}{3}))$	-4.369962354
$evalf(f(2\pi))$	-6.283185308

4. (10 pts) Suppose a function g satisfies all of the following properties. Sketch a graph of g that incorporates all of the following properties into it:

$$g(1) = -2 \quad g(2) = 2 \quad g(3) = 4$$

$$g'(1) = 0 \quad g'(3) = 0$$

$$g'(x) > 0 \text{ on } (1,3) \cup (3,\infty), \quad g'(x) < 0 \text{ on } (-\infty,1)$$

$$g''(x) > 0 \text{ on } (-\infty,2) \cup (3,\infty), \quad g''(x) < 0 \text{ on } (2,3)$$

5. (5 pts each) Evaluate the limits:

a. $\lim_{x \rightarrow \infty} (\sqrt{9x^2 + 3x + 7} - 3x)$

"Rationalize"
(a - b) (a + b)

$$\sqrt{9x^2 + 3x + 7} - 3x = \frac{\sqrt{9x^2 + 3x + 7} - 3x}{1} \cdot \frac{\sqrt{9x^2 + 3x + 7} + 3x}{\sqrt{9x^2 + 3x + 7} + 3x}$$

$$= \frac{(\sqrt{9x^2 + 3x + 7})^2 - (3x)^2}{\sqrt{9x^2 + 3x + 7} + 3x} = \frac{9x^2 + 3x + 7 - 9x^2}{\sqrt{9x^2 + 3x + 7} + 3x}$$

$$= \frac{3x \left(1 + \frac{7}{3x}\right)}{\sqrt{9x^2} \sqrt{1 + \frac{1}{3x} + \frac{7}{9x^2}} + 3x} = \frac{3x \left(1 + \frac{7}{3x}\right)}{3x \left(\sqrt{1 + \frac{1}{3x} + \frac{7}{9x^2}} + 1\right)}$$

$$\sqrt{9x^2} = \sqrt{9} \sqrt{x^2} = 3\sqrt{x^2} = 3|x| = 3x$$

$$\xrightarrow{x \rightarrow \infty} \frac{1 + 0}{\sqrt{1 + 0 + 0} + 1} = \frac{1}{\sqrt{1} + 1} = \frac{1}{2}$$

b. $\lim_{x \rightarrow \infty} \frac{3x^3 - 6x + 7}{2 - 5x - 7x^3} = \frac{3}{-7}$

6. (10 pts) Find the equation of the oblique asymptote for $f(x) = \frac{x^3 - 6x + 7}{x^2 - 5}$. This is sort of a limit at infinity.

$$\begin{array}{r}
 \textcircled{3x} \rightarrow y = 3x \\
 x^2 - 5 \overline{) 3x^3 + 0x^2 - 6x + 7} \\
 \underline{-(3x^3 \quad -15x)} \\
 \textcircled{9x + 7}
 \end{array}$$

$$\frac{3x^3}{x^2} = 3x$$

$$R(x) = 3x + \frac{9x+7}{x^2-5}$$

$$\frac{2B}{3} = 9 + \frac{1}{3}$$

7. (10 pts) Find the minimum vertical distance between $h(x) = 2x^2 - 5x + 12$ and $k(x) = 1 - 3x - x^2$.

$$f(x) = |2x^2 - 5x + 12 - (1 - 3x - x^2)| = \text{vertical Dist.}$$

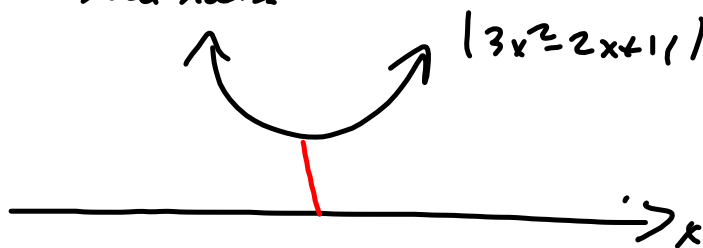
$$= |2x^2 - 5x + 12 + x^2 + 3x - 1|$$

$$= |3x^2 - 2x + 11| = \begin{cases} 3x^2 - 2x + 11 & \text{if } 3x^2 - 2x + 11 \geq 0 \\ -(3x^2 - 2x + 11) & \text{if } 3x^2 - 2x + 11 < 0 \end{cases}$$

$$b^2 - 4ac = (-2)^2 - 4(3)(11)$$

$$< 0 \rightarrow \text{No real roots.}$$

$$= 3x^2 - 2x + 11, \text{ always!}$$



$$f'(x) = 6x - 2 \stackrel{\text{SET}}{=} 0 \Rightarrow$$

$$6x = 2$$

$$x = \frac{1}{3} \text{ gives min distance.}$$

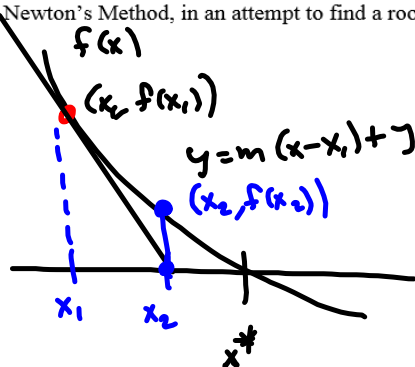
Min Distance

$$= f\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{3}\right) + 11$$

$$= \frac{1}{3} - \frac{2}{3} + \frac{32}{3}$$

$$= \frac{32}{3} = \text{min distance} \quad \textcircled{9} \quad x = \frac{1}{3}$$

8. (10 pts) Use the graph of the function $f(x)$, on the accompanying sheet, to show how x_2 would be found by Newton's Method, in an attempt to find a root. Derive the formula for x_2 , and explain what's going on.



Make a guess: x_1

Tangent line @ x_1 is

$$y = f'(x_1)(x - x_1) + f(x_1)$$

x_2 is where the tan line crosses the x-axis:

$$f'(x_1)(x - x_1) + f(x_1) = 0$$

Solve for x :

$$f'(x_1)(x - x_1) = -f(x_1)$$

$$x - x_1 = -\frac{f(x_1)}{f'(x_1)}$$

$$x = x_1 - \frac{f(x_1)}{f'(x_1)}$$

We want $x^* = \text{where } f(x) = 0$

$$f'(x_1)x - f'(x_1)x_1 = -f(x_1)$$

$$f'(x_1)x = f'(x_1)x_1 - f(x_1)$$

$$x = \frac{f'(x_1)x_1 - f(x_1)}{f'(x_1)}$$

$$x = x_1 - \frac{f(x_1)}{f'(x_1)} \equiv x_2$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

In general



9. (10 pts) Suppose $f''(x) = 40x^3 - 24x^2 + 18x - 2$, and we have the initial conditions $f'(1) = f(1) = 3$. Find $f(x)$.

$$\begin{aligned} \rightarrow f'(x) &= 40 \frac{x^4}{4} - 24 \frac{x^3}{3} + 18 \frac{x^2}{2} - 2x + C \\ &= 10x^4 - 8x^3 + 9x^2 - 2x + C \end{aligned}$$

$$f'(1) = 10 - 8 + 9 - 2 + C = 3$$

$$= C + 9 = 3$$

$$\rightarrow \boxed{C = -6}$$

$$\text{So } f'(x) = 10x^4 - 8x^3 + 9x^2 - 2x - 6$$

$$\rightarrow f(x) = 2x^5 - 2x^4 + 3x^3 - x^2 - 6x + D$$

$$\text{¶ } f(1) = 2 - 2 + 3 - 1 - 6 + D = 3$$

$$\rightarrow D - 4 = 3$$

$$D = 7$$

$$\rightarrow \boxed{f(x) = 2x^5 - 2x^4 + 3x^3 - x^2 - 6x + 7}$$

